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Imaginary part of the active neutrino self-energy

M. Laine (AEC, ITP, University of Bern)

Results are presented for the imaginary part of the active neutrino self-energy matrix within the Standard Model of particle physics, originating from $2 \leftrightarrow 2$ and $1 \leftrightarrow 3$ scatterings described by the Fermi model at temperatures between 1 MeV and 10 GeV.[1] In this temperature range, the particle content of a Standard Model plasma changes significantly because of the presence of a QCD crossover. A way to account for hadronic effects in this regime was worked out in ref. [1]. An approximate phenomenological implementation was proposed in ref. [2].[2] The results play a role for the determination of the sterile neutrino production rate through active-sterile transitions, both at zero [1, 2] and at non-zero [3] lepton chemical potential.

Given that weak interactions of Standard Model neutrinos take place through vertices with the Dirac structure $\sim \gamma^\mu a_{\rm L}$, where $a_{\rm L} \equiv (1-\gamma_5)/2$ is a chiral projector, their inverse propagator is of the form [4]

$$S^{-1}(Q) = a_{\rm R}\,(\not{Q} + \not{\Sigma}\,)\,a_{\rm L}\;. \tag{1}$$

Because of the chiral projectors, the (retarded) self-energy can be expressed as

$$\not{\Sigma} = a\,\not{Q} + b\,\not{u}\;, \tag{2}$$

where $u \equiv (1,\mathbf{0})$ is the plasma four-velocity, and a, b are complex functions. The factor a represents a small correction to the tree-level term $\not{Q}$ and is conventionally omitted [5]. The coefficient b can be expressed as

$$b = b_r + i\,\frac{\Gamma}{2}\;, \tag{3}$$

where b_r represents a modification of the dispersion relation, and is often referred to as an "effective potential". As discussed in ref. [5], it gets generated at 1-loop level and is significantly modified if a lepton asymmetry is present. The topic of the present note is the imaginary part, Γ, which may be referred to as a "thermal width" or a "damping coefficient".

It is easy to see that the width, as defined by eq. (3), can be projected out as

$$\Gamma = \mathrm{Tr}\left\{\frac{\omega\,\not{Q} - Q^2\,\not{u}}{\mathbf{q}^2}\,\mathrm{Im}\,\not{\Sigma}\,a_{\rm L}\right\}\;, \quad Q \equiv (\omega,\mathbf{q})\;. \tag{4}$$

[1] At temperatures above a few GeV, $1 \leftrightarrow 2$ processes overtake the Fermi scatterings but have not been included in the plots here. To see their effect, please consult the data files on the web page.

[2] Please consult the erratum of this paper for a few technical corrections.

If we introduce the objects

$$I_Q \;\equiv\; \mathrm{Tr}\,\{\not{Q}\;\mathrm{Im}\,\not{\Sigma}\,a_{\rm L}\}\;, \quad I_u \;\equiv\; \mathrm{Tr}\,\{\,\not{u}\;\mathrm{Im}\,\not{\Sigma}\,a_{\rm L}\}\;, \tag{5}$$

then $\Gamma = (\omega\, I_Q - Q^2\, I_u)/\mathbf{q}^2$. In the following we assume Γ to be evaluated at the on-shell point of sterile neutrinos, with $\omega \equiv \sqrt{\mathbf{q}^2 + M_1^2}$.

For a numerical evaluation of the objects in eq. (5), it is natural to factor out the combination $G_{\rm F}^2 T^4$ (here $G_{\rm F}$ denotes the Fermi constant) as well as appropriate powers of ω:

$$\hat{I}_Q \;\equiv\; \frac{I_Q}{G_{\rm F}^2\, T^4\, \omega^2}\;, \qquad \hat{I}_u \;\equiv\; \frac{I_u}{G_{\rm F}^2\, T^4\, \omega}\;. \tag{6}$$

With this notation,

$$\Gamma(\mathbf{q}) = G_{\rm F}^2\, T^4\, \omega \left\{ \hat{I}_Q + \frac{M_1^2}{\mathbf{q}^2}\Big(\hat{I}_Q - \hat{I}_u\Big) \right\}_{\omega=\sqrt{\mathbf{q}^2+M_1^2}}\;. \tag{7}$$

The latter term is of subleading importance if $M_1 \ll |\mathbf{q}| \sim 3T$. Numerical results for $\hat{I}_Q$ and $\hat{I}_u$, for different $\mathbf{q}$ and charged lepton flavours α, are shown in figs. 1–6 (for these results the bottom quark has been added to Table 1 of ref. [2], and we have set $M_1 = 10$ keV).

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References

[1] T. Asaka, M. Laine and M. Shaposhnikov, *On the hadronic contribution to sterile neutrino production,* JHEP 06 (2006) 053 [hep-ph/0605209].

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[3] M. Laine and M. Shaposhnikov, *Sterile neutrino dark matter as a consequence of νMSM-induced lepton asymmetry,* JCAP 06 (2008) 031 [0804.4543].

[4] H.A. Weldon, *Effective fermion masses of $\mathcal{O}(gT)$ in high-temperature gauge theories with exact chiral invariance,* Phys. Rev. D 26 (1982) 2789.

[5] D. Nötzold and G. Raffelt, *Neutrino Dispersion at Finite Temperature and Density,* Nucl. Phys. B 307 (1988) 924.

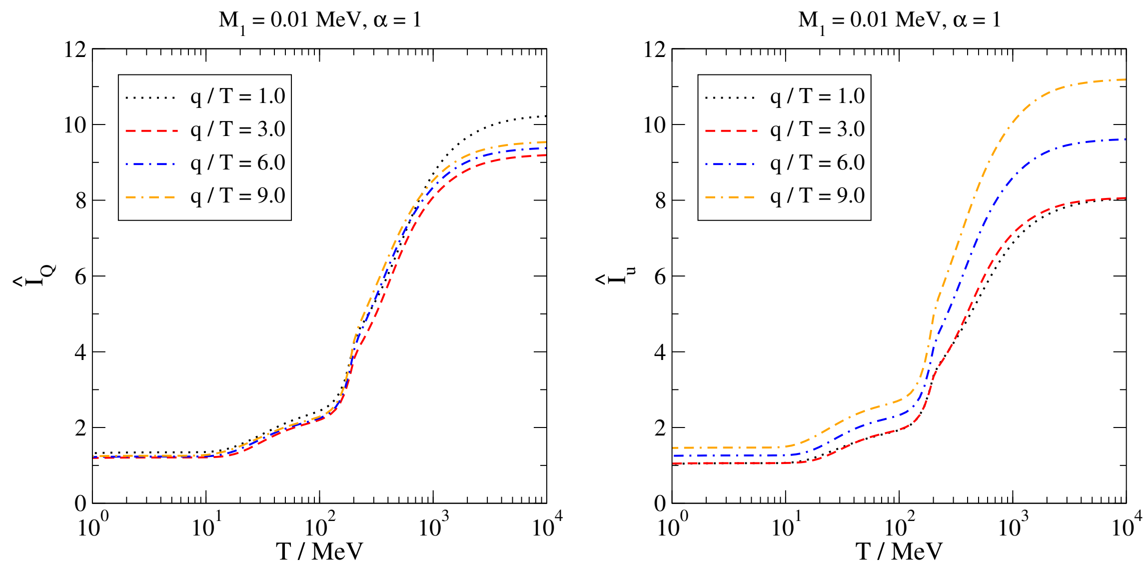

Figure 1: The quantities $\hat{I}_Q$ (left) and $\hat{I}_u$ (right), defined in eq. (6), for $\alpha = 1$.

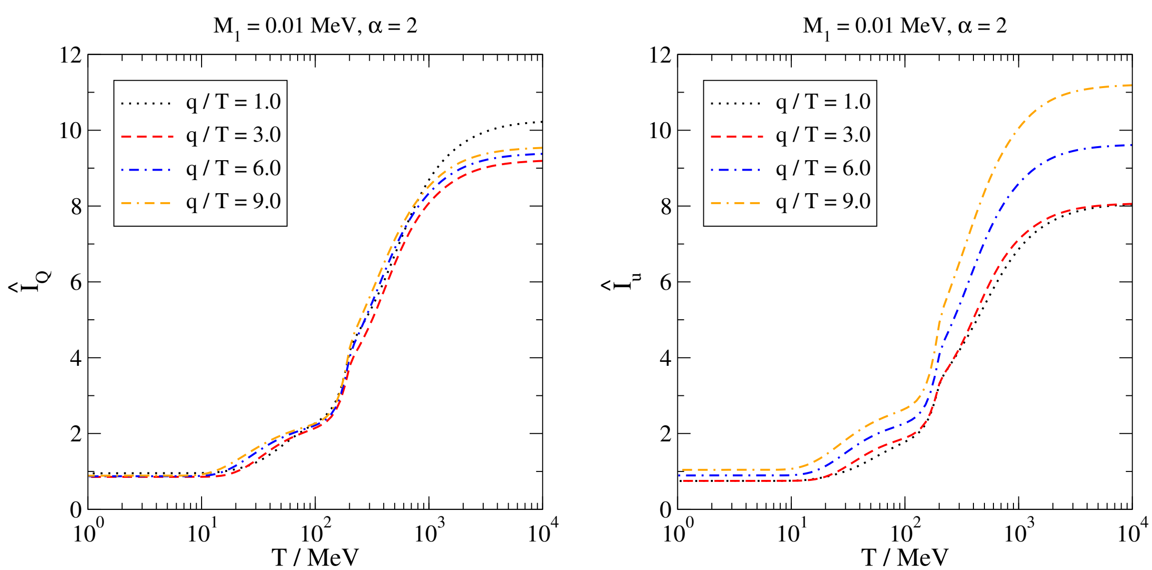

Figure 2: The quantities $\hat{I}_Q$ (left) and $\hat{I}_u$ (right), defined in eq. (6), for $\alpha = 2$.

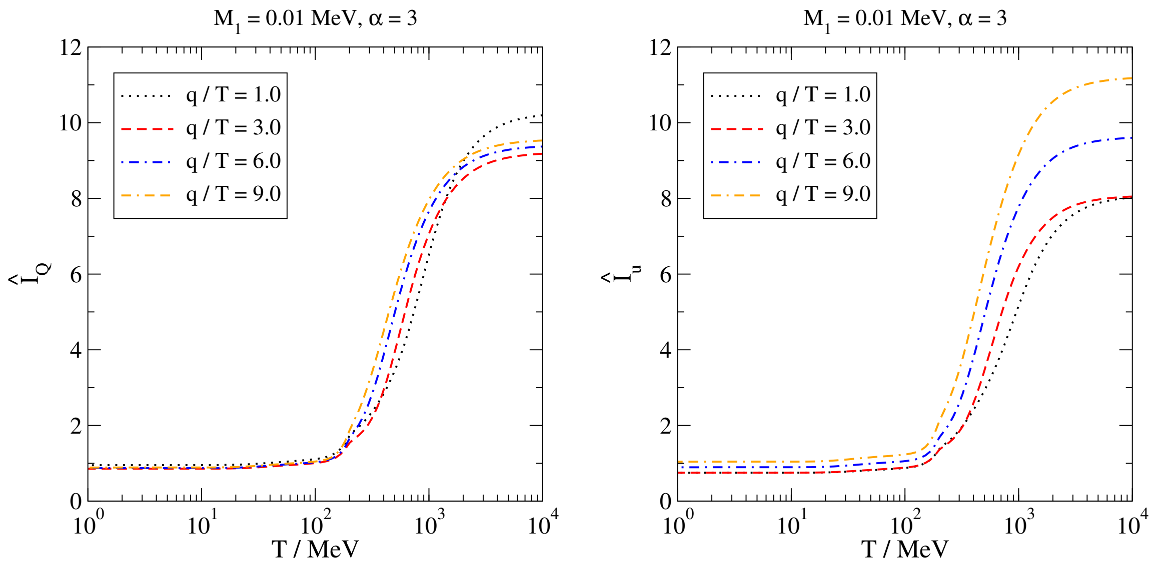

Figure 3: The quantities $\hat{I}_Q$ (left) and $\hat{I}_u$ (right), defined in eq. (6), for $\alpha = 3$.

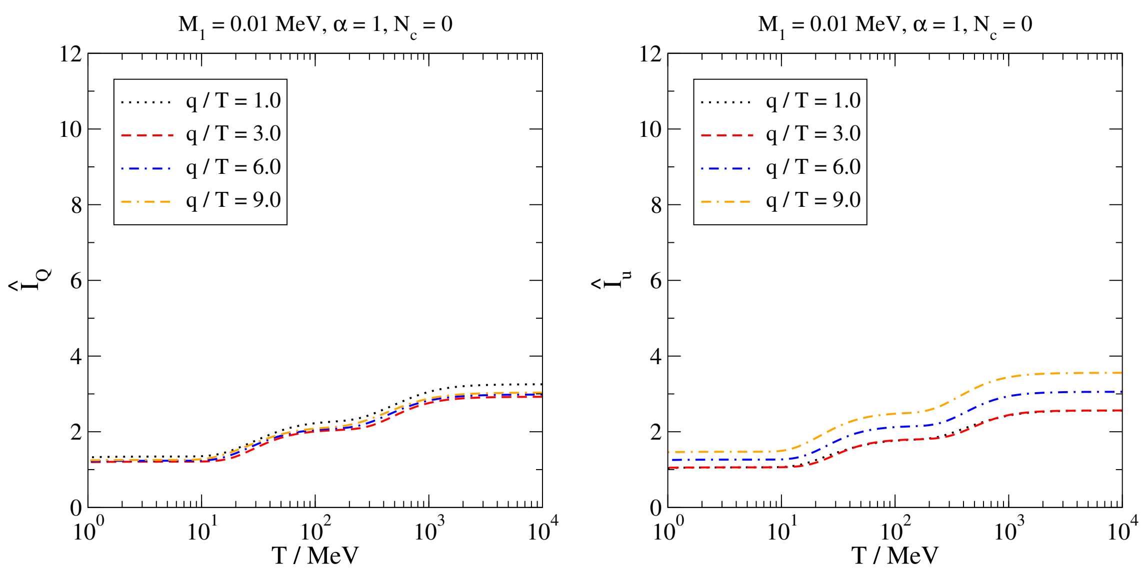

Figure 4: Like fig. 1 but with hadronic effects omitted ($N_c = 0$).

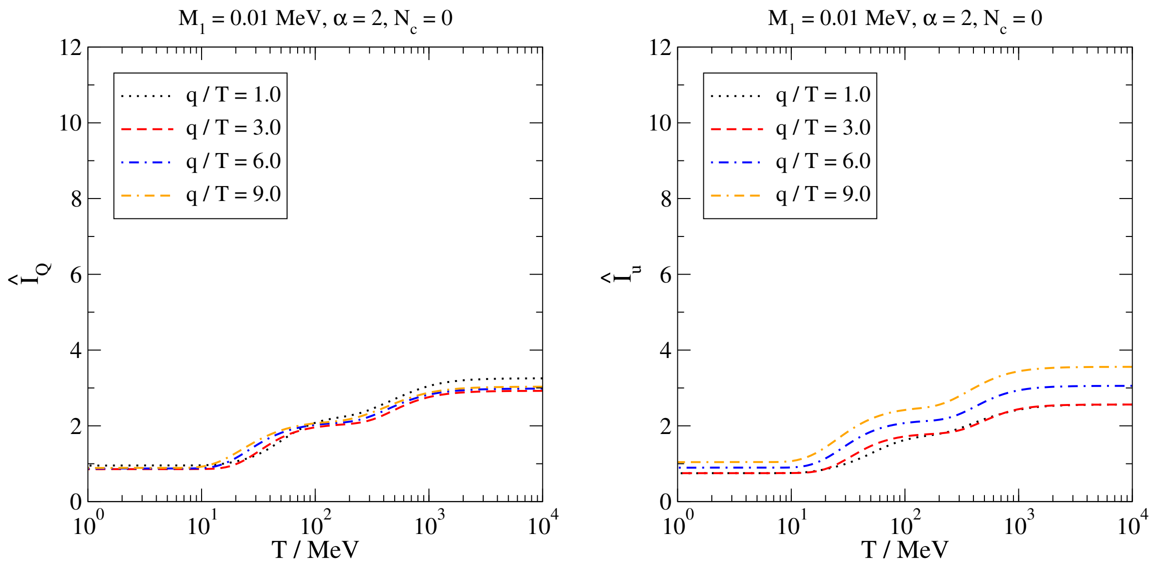

Figure 5: Like fig. 2 but with hadronic effects omitted ($N_{\rm c} = 0$).

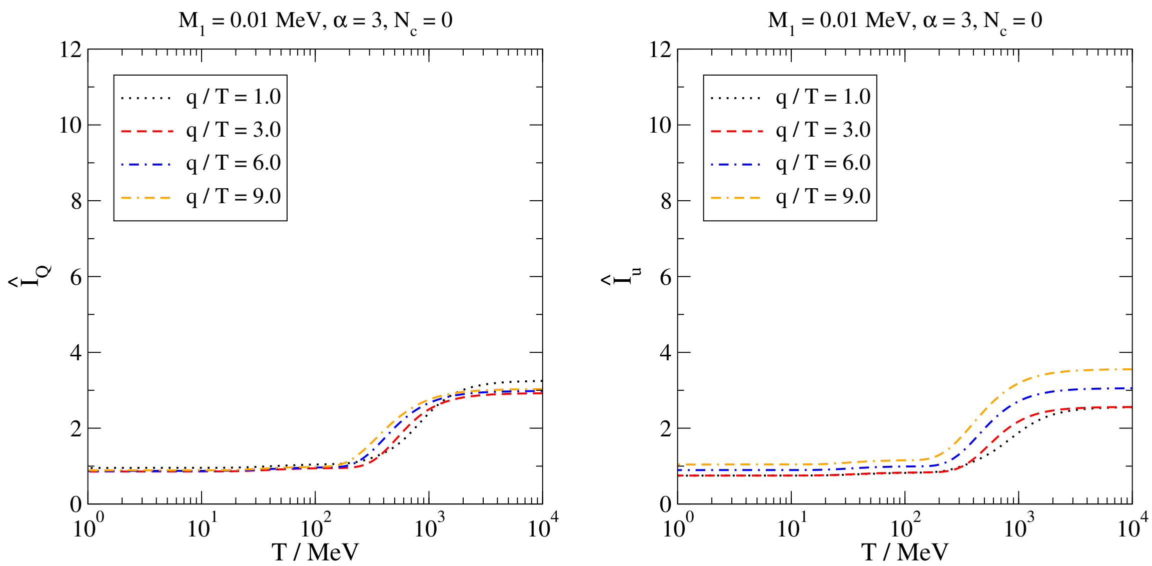

Figure 6: Like fig. 3 but with hadronic effects omitted ($N_{\rm c} = 0$).