The effective theory approach to hot QCD thermodynamics

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The goal: a simple method for determining physical observables with good theoretical and numerical precision, at temperatures $2T_c^{\text{QCD}} \lesssim T \lesssim T_c^{\text{EW}}$ ....

.... with the hope that one day finite temperature QCD can be systematically studied with collider experiments, or that the tools developed find applications in cosmology.

To start with: static observables. May allow, however, for $N_f > 0$ light dynamical quarks and a baryon chemical potential.

(For brevity, the equations shown below are for $N_f = 0$.)

In this setting, perturbation theory has a number of problems.

Expansion parameter related to bosonic fluctuations with momentum scale p:

$$\epsilon \sim \frac{1}{\pi} g^2(2\pi T) n_b(p) = \frac{g^2(2\pi T)}{\pi(e^{p/T}-1)} \stackrel{p<T}{\sim} \frac{g^2(2\pi T)T}{\pi p}.$$

For hard modes, $p \sim 2\pi T$, expansion is $\sim g^2/\pi^2$ like at $T=0$, and should remain viable down to a scale ~ 2 GeV $\Leftrightarrow \sim 2T_c$.

For colour-electric modes, $p \sim gT$, the series is only in $\sim g/\pi$, and seems to converge much worse, unless $T \gg T_c$.

Braaten, Nieto; ...

For colour-magnetic modes, $p \sim g^2 T$, no perturbative series.

Linde;
Gross, Pisarski, Yaffe

Thus, there is certainly one simplification from the full theory one can make: at $T \gtrsim 2T_c$, the scales $p \sim 2\pi T$ can be integrated out, to obtain a dimensionally reduced effective theory for the bosonic zero modes:

$$\mathcal{L}_E = \frac{1}{2} \mathrm{Tr}\, F_{ij}^2 + \mathrm{Tr}[D_i, A_0]^2 + m_D^2 \,\mathrm{Tr}\, A_0^2 + \lambda_A (\mathrm{Tr}\, A_0^2)^2.$$

Here $m_D^2 \sim g^2 T^2$, $\lambda_A \sim g^4 T$ are generated radiatively.

Ginsparg; Appelquist, Pisarski; ...

But can one go any further? I.e., how serious is the convergence problem related to the scale $p \sim gT$, represented by A_0?

For the sake of this talk, let us make a conjecture:

If quantities are determined up to and including the leading non-perturbative constant, such that effects from all the scales have made their entrances, then things should already be OK!

Concrete examples to be discussed in the following:

- Spatial string tension $\sqrt{\sigma_s}$;
  Correlation lengths $\xi_G \equiv m_G^{-1}$ related to $\sim \mathrm{Tr}\, F_{ij}^2$:

$$\sqrt{\sigma_s}, m_G \sim \text{[non-pert]} \times g^2 T + \ldots \, .$$

- Correlation lengths $\xi_E^{(i)} \equiv [m_E^{-1}]^{(i)}$ for $\mathrm{Re}\,\mathrm{Tr}$ Pol, $\mathrm{Im}\,\mathrm{Tr}$ Pol, $\mathrm{Tr}\, F_{0i} F_{jk}$, etc:

$$m_E \sim gT + g^2 T (\ln \frac{1}{g} + \text{[non-pert]}) + \ldots \, .$$

- Pressure, energy density, ...:

$$\frac{p}{T^4} \sim 1 + g^2 + g^3 + g^4 \ln \frac{1}{g} + g^5 + g^6 (\ln \frac{1}{g} + \text{[non-pert]}) + \ldots \, .$$

Some evidence for the conjecture comes from $\sqrt{\sigma_s}$, $[m_E]^{(i)}$!

To determine the series up to the non-perturbative part, need to systematically integrate out A_0 to a sufficient depth, and measure the remaining numerical factor, using the theory

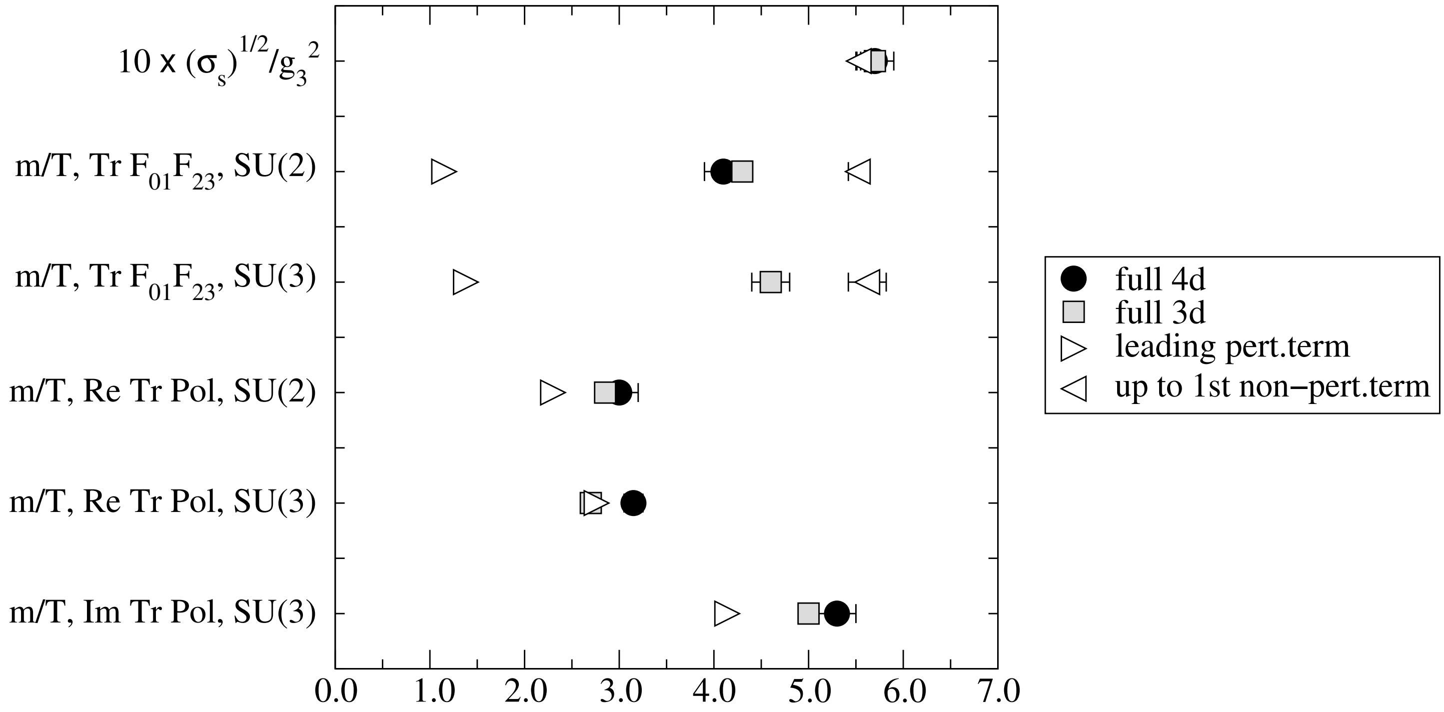
$$\mathcal{L}_M = \frac{1}{2} \mathrm{Tr}\, F_{ij}^2 \; .$$

- For $\sqrt{\sigma_s}$ one only needs to compute the change in the effective coupling constant $g_3^2 = g^2 T$.
- For $[m_E]^{(i)}$ one needs explicit matching coefficients:

$$\begin{aligned}
[m_E]^{(i)} &= -\lim_{z\to\infty} \frac{1}{z} \ln \langle \mathcal{O}_E^{(i)}(z) \mathcal{O}_E^{(i)}(0) \rangle_{\mathcal{L}_E} \\
&= gT + g^2 T \ln \frac{gT}{\Lambda_M} + \mathcal{O}(g^3 T) \\
&\quad - \lim_{z\to\infty} \frac{1}{z} \ln \langle \mathcal{Q}_M^{(i)}(z;0) \rangle_{\mathcal{L}_M, \Lambda_M} .
\end{aligned}$$

Arnold, Yaffe

Numerical results:



Lattice data from:

Karsch et al, hep-lat/9602007;
Teper, hep-lat/9804008;
Datta and Gupta, hep-lat/9906023;
Kaczmarek et al, hep-lat/9908010;
Hart and Philipsen, hep-lat/9908041;
Hart et al, hep-ph/0004060;
Datta and Gupta, hep-lat/0208001

For $p(T)$ one can carry out a similar matching computation.

- The first part, dimensional reduction, leads to

$$\begin{aligned} \frac{p}{T^4} &= 1 + g^2 + g^4 + g^6 + \dots + \frac{p_E}{T^3}, \\ \frac{p_E}{T^3} &= \frac{1}{VT^3} \ln \int \mathcal{D}A_i \mathcal{D}A_0 e^{-\int d^3x \mathcal{L}_E}. \end{aligned}$$

- The second part, integration over A_0, leads to

$$\begin{aligned} \frac{p_E}{T^3} &= g^3 + g^4 + g^5 + g^6 + \dots + \frac{p_M}{T^3}, \\ \frac{p_M}{T^3} &= \frac{1}{VT^3} \ln \int \mathcal{D}A_i e^{-\int d^3x \mathcal{L}_M}. \end{aligned}$$

Coefficients determined up to order $\mathcal{O}(g^5)$ by Braaten, Nieto, but this does not yet allow to resolve the non-perturbative part.

To reach the level g^6, 4-loop matching coefficients are needed.

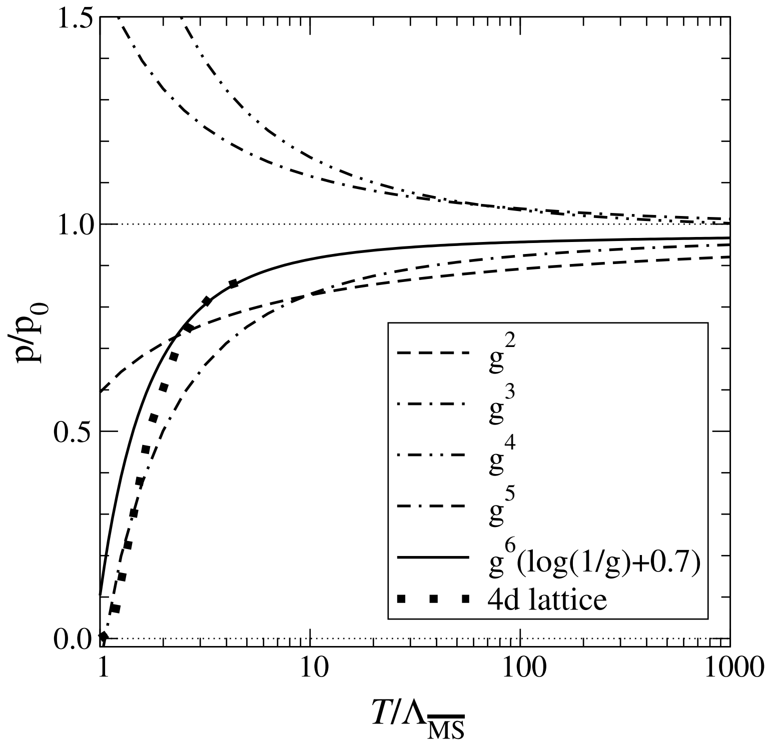
We ($\approx$ Y. Schröder) have recently finished a computation of the coefficient of the logarithmic term.

K.Kajantie, M.Laine, K.Rummukainen, Y.Schröder, in preparation

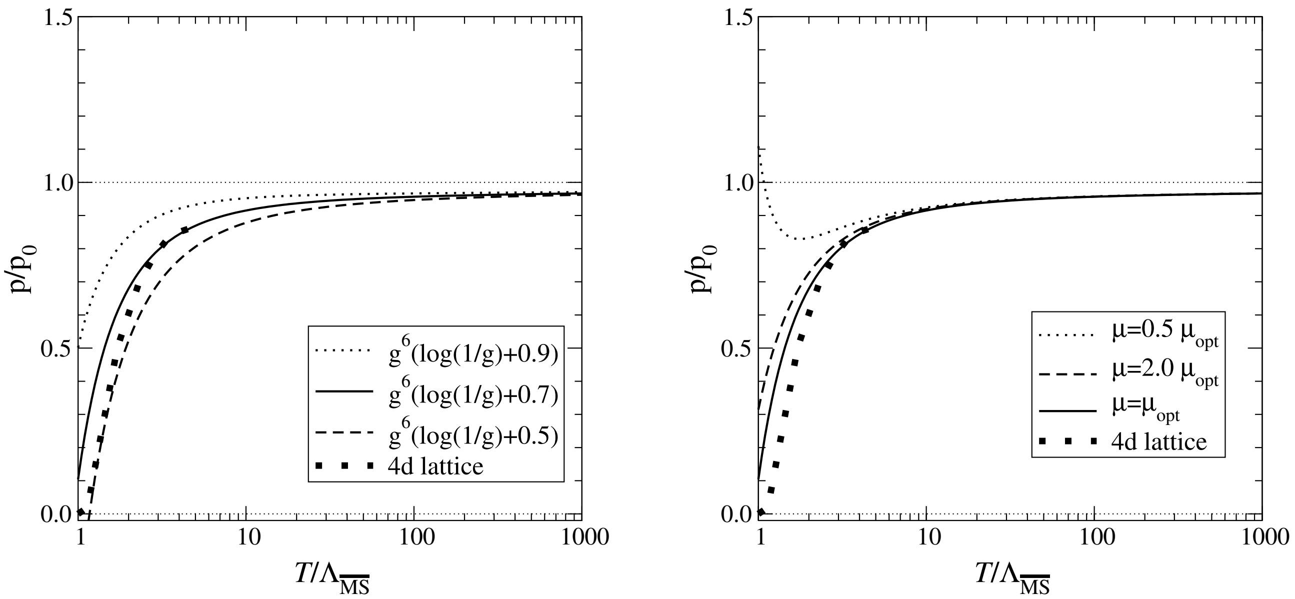
The constant part is of course missing. Unfortunately it includes contributions from all the scales $(2\pi T, gT, g^2T)$, which makes its determination at least an order of magnitude harder.

However, treating it as a free parameter, one can already compare with 4d lattice data, to see if the conjecture has any chance of surviving.

The outcome for a particular constant, employing the $\overline{\mathrm{MS}}$ scale $\bar{\mu}_{\,\mathrm{opt}} \approx 2\pi T$ suggested by dimensional reduction:



Dependence on the constant (left) and on $\bar{\mu}$ (right).



Conclusions

There is some evidence for the conjecture from correlation lengths, but more quantum number channels should be added.

The conjecture might also work for the pressure, although there is a lot of work remaining before we can know for sure.

Once such analytic expressions with non-perturbative constants are known, extensions to finite $N_f > 0$ and $\mu \lesssim \pi T$ are easy.

Hopefully similar expressions can be worked out also for real time observables (cf. sphaleron rate), where 4d lattice simulations are even more difficult than in the static case.