Q-ball Thermodynamics

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(*Much influenced by A.Kusenko*)

1. **Bose-Einstein Condensation in Relativistic Field Theory**
2. **Instability and Q-balls**
3. **The Significance of All This for Particle Physics and Cosmology**
4. **Conclusions**

Consider the field theory of one complex scalar field:

$$\mathcal{L}_E = \partial_\tau \phi^* \partial_\tau \phi + \sum_{i=1}^3 \partial_i \phi^* \partial_i \phi + m^2 \phi^* \phi + \lambda(\phi^*\phi)^2,$$

$$Z = \int \mathcal{D}\phi \exp(-\int d\tau d^3x \mathcal{L}_E).$$

What is the phase diagram for finite T, μ?

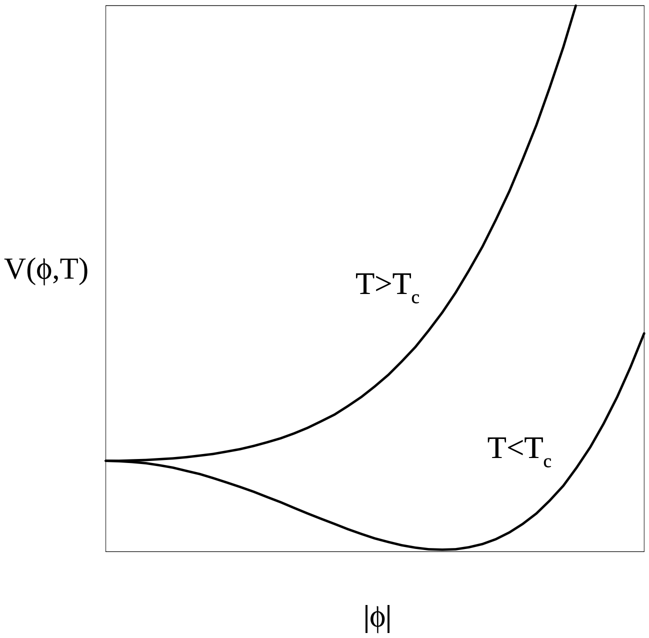
First, just heat up the system!

$$Z = \exp(-\beta F) = \mathrm{Tr}\exp(-\beta H), \quad \beta = \frac{1}{T}.$$

[Finite temperature in field theory: periodic boundary conditions in the path integral, $\phi(0, \mathbf{x}) = \phi(\beta, \mathbf{x})$.]

The phase structure can be studied with an effective potential $=$ the free energy density with a homogeneous condensate:

$$\begin{aligned} f(T) &= \frac{1}{V} F(T, V) = V(\phi, T)|_{\partial_\phi V = 0}, \\ V(\phi, T) &\sim (m^2 + \frac{1}{3}\lambda T^2)\phi^*\phi + \lambda(\phi^*\phi)^2. \end{aligned}$$



For $m^2 < 0$, the symmetry is broken at $T = 0$, but it is restored at high temperatures, just like in, e.g., a ferromagnet!

Another direction in the phase diagram: **a conserved charge!**

$$Q = i\int d^3x(\phi^*\partial_t\phi - \phi\partial_t\phi^*).$$

Classical equations of motion $\Rightarrow$ Q is a constant.

The conserved charge Q corresponds to:

$\star$ particle number in non-relativistic theories (but can be < 0)
$\star$ baryon (B) or lepton (L) number in field theories.

(Note: electric charge is a bit different!)

To Q, one can associate a chemical potential μ:

$$Z = \exp[-\beta\Omega(T,V,\mu)] = \mathrm{Tr}\exp[-\beta(H-\mu Q)].$$

[In field theory: $\partial_\tau\phi^*\partial_\tau\phi \Rightarrow (\partial_\tau+\mu)\phi^*(\partial_\tau-\mu)\phi$.]

In cosmology, the physical quantity is the charge density,

$$n_Q = \frac{Q}{V} = -\frac{\partial V(\phi,T,\mu)}{\partial\mu}.$$

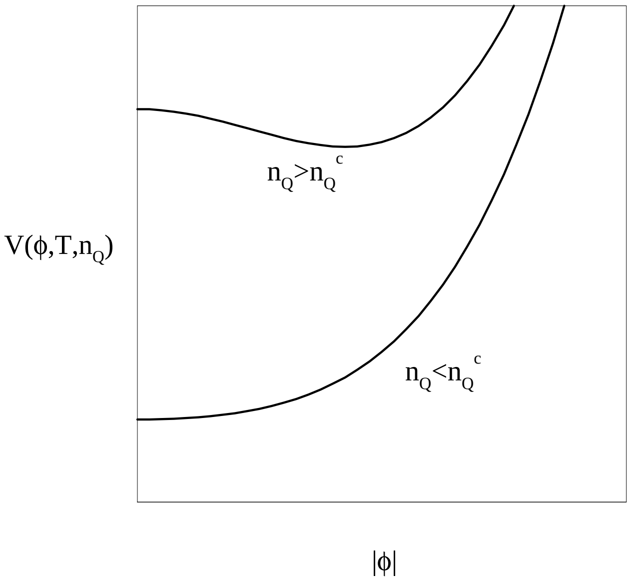
$\Rightarrow$ make a Legendre transformation:

$$V(\phi,T,n_Q) = V(\phi,T,\mu) + \mu n_Q.$$

For the previous theory:

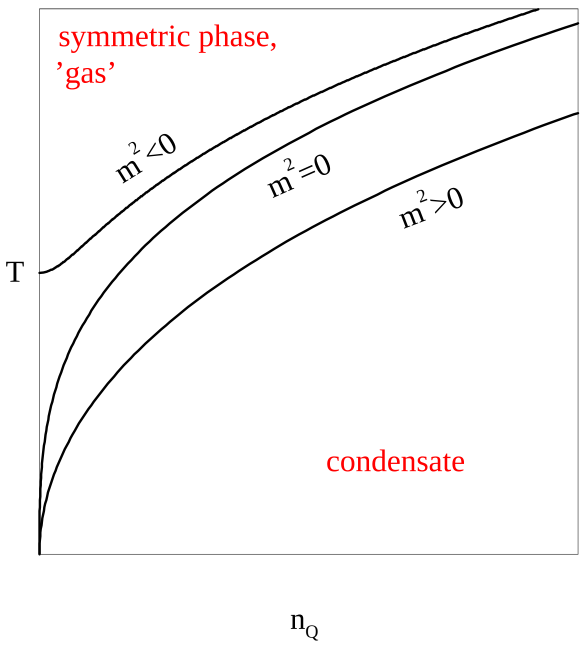
$$V(\phi,T,\mu) \sim -\tfrac{1}{6}\mu^2T^2 + (m^2-\mu^2+\tfrac{1}{3}\lambda T^2)\phi^*\phi + \lambda(\phi^*\phi)^2,$$
$$V(\phi,T,n_Q) \sim \frac{n_Q^2/4}{T^2/6+\phi^*\phi} + (m^2+\tfrac{1}{3}\lambda T^2)\phi^*\phi + \lambda(\phi^*\phi)^2.$$

$\Rightarrow$ a chemical potential favours condensation!



Take a fixed T,
and increase n_Q
$\Rightarrow$ **Bose-Einstein condensation!**

The complete phase diagram:



The next question: is the condensate stable?

Ways to study it:

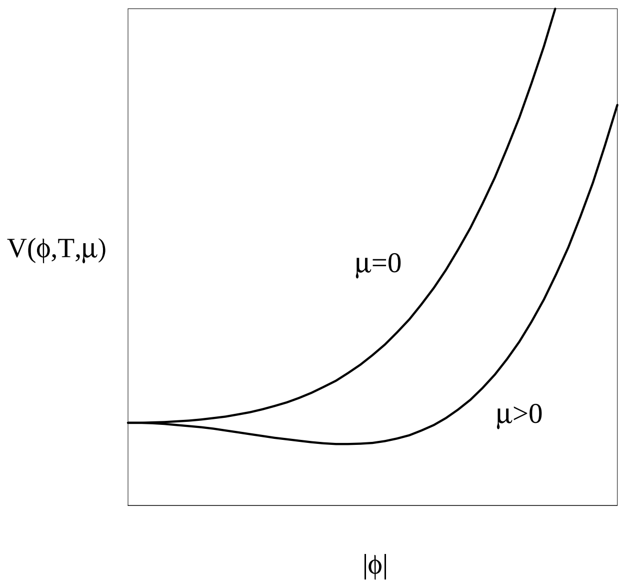
⋆ the entropy of any subsystem must be at a maximum with respect to fluctuations of the particle number ⇒

$$\left(\frac{\partial \mu}{\partial n_Q}\right)_T = \left(\frac{\partial^2 f}{\partial n_Q^2}\right)_T > 0.$$

⋆ the fluctuation matrix around the saddle point has only positive eigenvalues ⇒

$$\left.\frac{\partial^2 V(\phi, T, \mu)}{\partial \phi^2}\right|_{\phi=\phi_{\text{min}}} > 0.$$

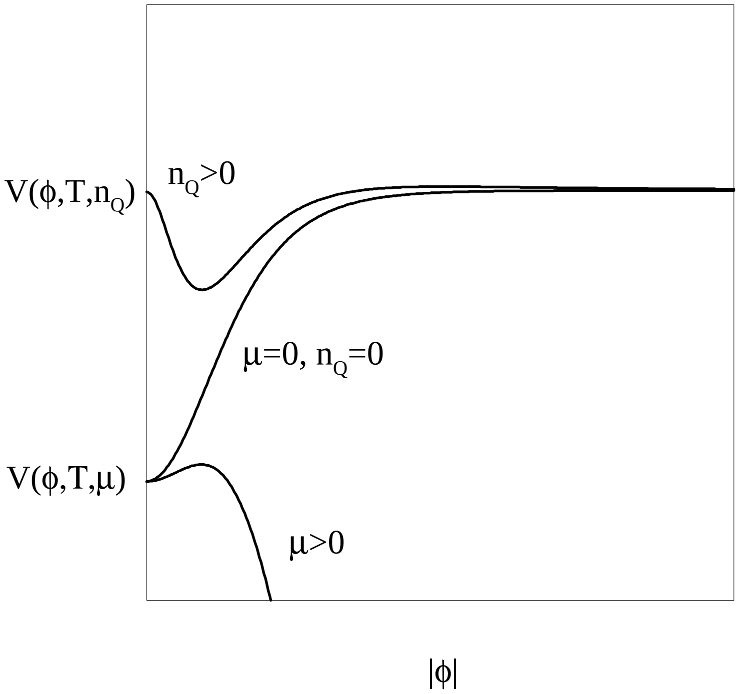
For the potential $m^2\phi^*\phi + \lambda(\phi^*\phi)^2$:



⇒ **the condensate is (meta)stable!**

However, there are cases where the condensate is unstable!

Consider, in particular, theories with **a flat potential**
($=$ a potential which rises more slowly than $\propto |\phi|^2$):



A condensate could exist
[since $V(\phi, T, n_Q)$ has a minimum at $|\phi| > 0$]
but it is unstable
[since $V(\phi, T, \mu)$ is at a maximum]:

$$\left.\frac{\partial^2 V(\phi, T, \mu)}{\partial \phi^2}\right|_{\phi=\phi_{\text{min}}} < 0.$$

What happens in the unstable system?

One can have an inhomogeneous ground state!

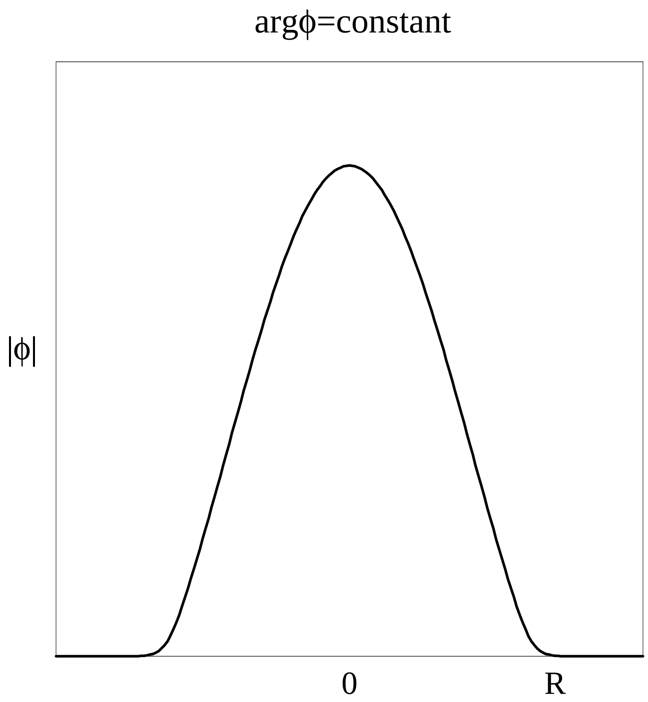
Look for (static) spherically symmetric extrema of

$$F(T,\mu) \sim \int d^3x \left[\partial_i\phi^*\partial_i\phi - \mu^2\phi^*\phi + V(\phi)\right],$$

with the boundary conditions $\phi'(0)=0, \phi(\infty)=0$.

The solution in a flat potential:

A Q-ball
(a non-topological soliton)
[Not a thin-wall object!]

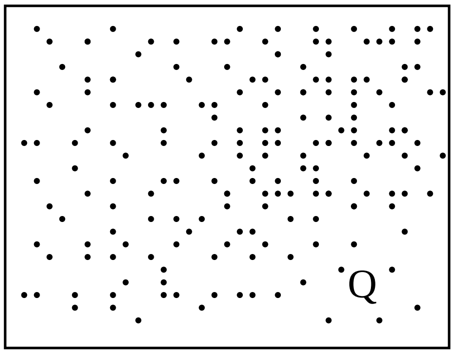


If $\quad V(\phi) \stackrel{|\phi|\to\infty}{\longrightarrow} M(T)^4, \quad M(T)^4 \sim M^4 + NT^4,$ then

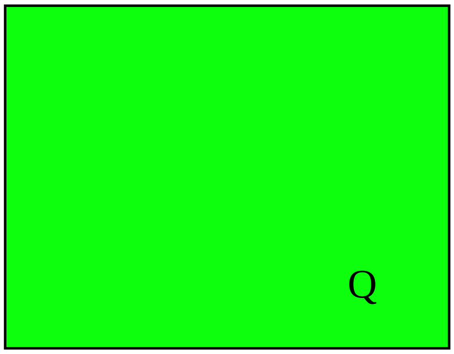
$$R \approx \frac{1}{\sqrt{2}} M(T)^{-1} Q^{1/4},$$

$$F(T,Q) \approx \frac{4\pi\sqrt{2}}{3} M(T)\, Q^{3/4}.$$

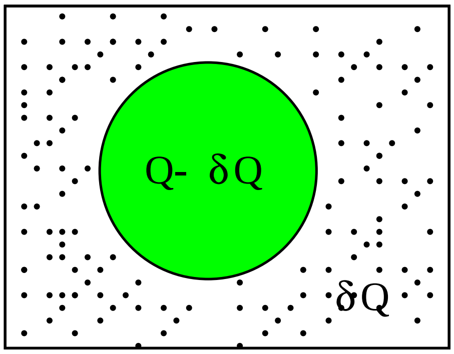
The possible phases of the system at finite T, n_Q:



Metastable!
$F_Q = F_0 + \frac{Q^2}{VT^2}$
$\mu = \frac{Q}{VT^2}$



Unstable!



Metastable!
$F_Q = F_0 + M(T)(Q - \delta Q)^{3/4}$
$\mu = \frac{\delta Q}{VT^2} = M(T)(Q - \delta Q)^{-1/4}$

Either the first or the last is the ground state: the last if

$$V \lesssim \frac{Q^{5/4}}{M(T)T^2}.$$

If n_Q is fixed, then Q-balls are stable at $V \to \infty$!

The previous discussion was at $T > 0$. At $T = 0$ one needs to compare the energies!

⋆ Q-ball: $E_Q \sim MQ^{3/4}$

⋆ Free particles: $E_{\text{free}} \sim mQ$, where m is the mass of the lightest charged particle.

Thus, Q-balls are stable if $E_Q < E_{\text{free}}$, which gives a lower bound for the charge.

For instance, for "baryonic Q-balls", m is the proton mass $m_p \approx 1$ GeV and Q-balls are stable if

$$Q > 10^{15}(M/1\mathrm{TeV})^4.$$

"Leptonic Q-balls", in contrast, are not stable since neutrinos are massless!

The conclusion: if there is

⋆ a conserved global charge,

⋆ a charged scalar field with a flat potential,

⋆ only massive charged particles,

then Q-balls with a large charge are absolutely stable at zero temperature, and can constitute the ground state of the system also at finite temperatures!

Open questions:

⋆ Are the properties required realized anywhere in Nature?	Perhaps!
⋆ Can Q-balls with large charges be actually formed in the cosmological background?	Perhaps!
⋆ If formed, do such Q-balls reach thermodynamical equilibrium?	Thermal yes, chemical no!
⋆ Do the Q-balls survive thermalization?	Yes!
⋆ Can such Q-balls have physical consequences and be experimentally detectable?	Yes!

[⋆ In case Q-balls are unstable (no flat directions), could they still have implications? Enqvist, McDonald]

Can Q-balls be realized in the Nature?

Not in the Standard Model!

The particle content:

⋆ gauge bosons
(photon, $W^{\pm}$, Z^0, gluons)

⋆ Higgs the only scalar!

⋆ leptons
(electron, muon, tau, neutrinos)

⋆ quarks

However:

⋆ the potential of the Higgs field is not flat
$[V = -(m_H^2/2)\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2]$

⋆ the Higgs field is not charged under B, L

$\Rightarrow$ **there are no Q-balls!**

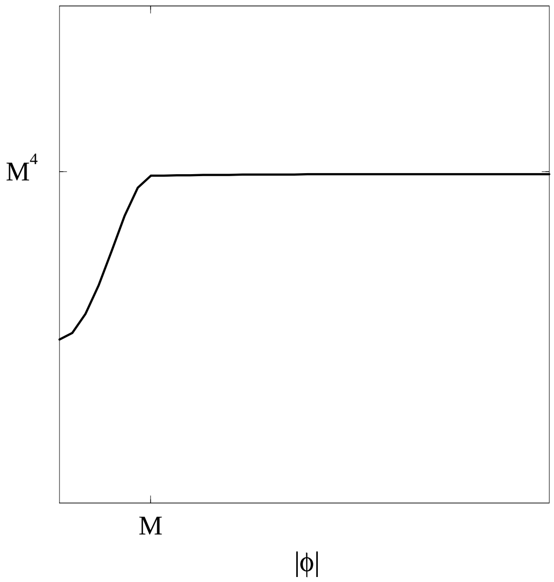
In Supersymm. extensions of the Standard Model, things are different: there is a superpartner for each of the SM particles!

gauge bosons (photon, $W^{\pm}$,...)	$\Rightarrow$ gauginos	fermions, not charged
Higgs	$\Rightarrow$ Higgsino	
leptons (electron, muon,...)	$\Rightarrow$ slepton	scalars, charged! (B, L)
quarks	$\Rightarrow$ squarks	

1. Thus there are charged scalars.

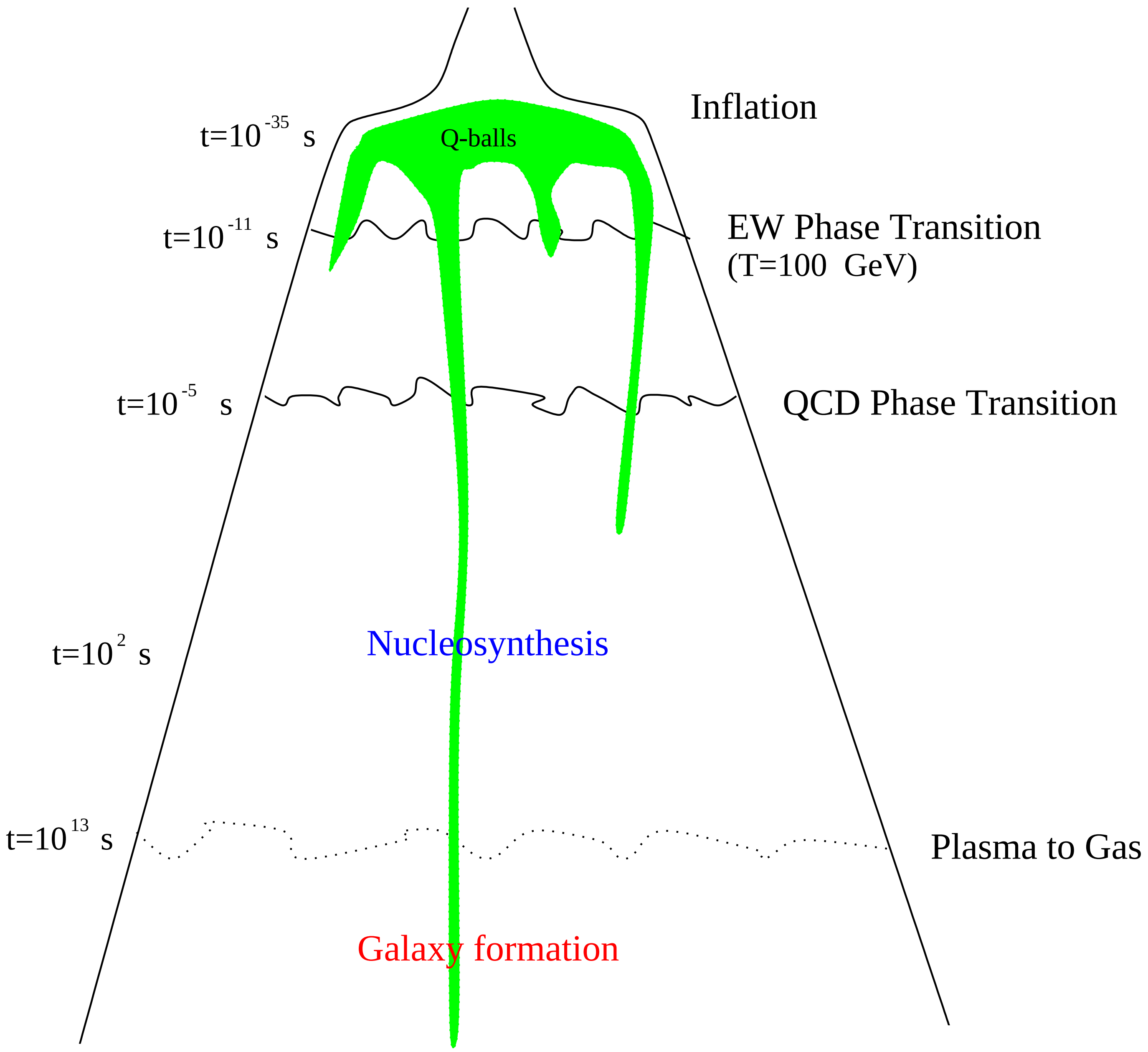
2. Moreover, in unbroken SUSY, there are a lot of flat directions for these fields!

   In the realistic broken SUSYs, the flat directions are lifted. However, in the so called gauge-mediated scenarios for SUSY breaking, the directions remain flat for large field values!



3. All baryons are massive.

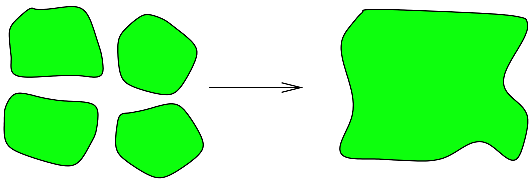
The cosmological background:



Can one form Q-balls with large charges $Q \gtrsim 10^{20}$?

One possible mechanism: accretion.

Generate small Q-balls with fluctuations and then combine to make them larger.



However, it is difficult to get large enough charges this way!

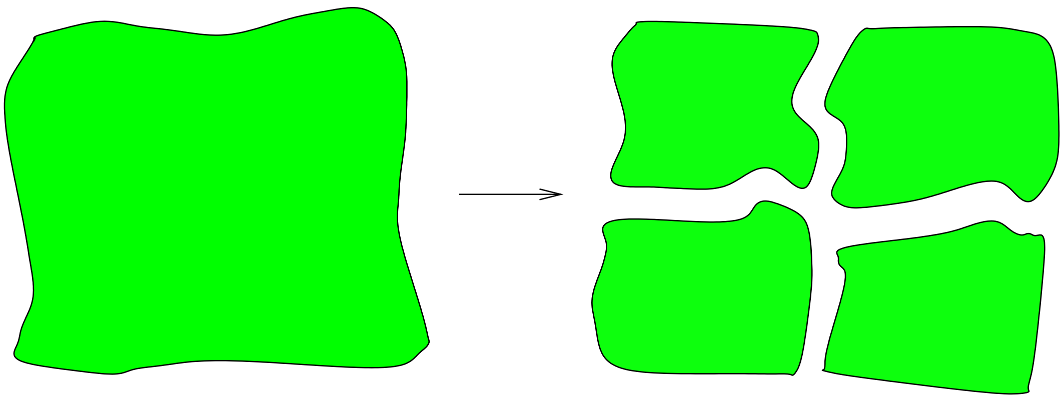
On the other hand, Q-balls with larger charges can perhaps be produced with the **Affleck-Dine mechanism**.

The idea: the energy density during inflation is $\sim (10^{15}\text{GeV})^4$. On the other hand, the energy density of a scalar condensate in a flat direction is $\sim (10^{3}\text{GeV})^4$. It is thus natural that a scalar condensate is formed as a result of fluctuations.

Moreover, the condensate may get charged: the GUTs relevant at the time of inflation are expected to involve baryon number violation and CP-violation, and inflation provides non-equilibrium.

$\Rightarrow$ **It is "natural" that there is a (baryonically) charged Bose-Einstein condensate in the Universe after inflation!**

However, as we have seen, a condensate is unstable!



Estimates of the instability formation show that

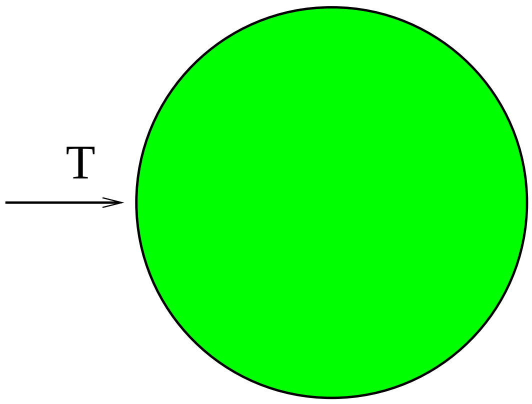
Kusenko, Shaposhnikov;
Enqvist, McDonald

- ⋆ the instability occurs at a sharply peaked macroscopic distance scale, typical of a Q-ball size,
- ⋆ the charge distribution of the decay products is also sharply peaked, and can be at $Q \gtrsim 10^{20}$,
- ⋆ most of the charge can end up in Q-balls ($\delta Q/Q \sim Q^{-1/4}$).

$\Rightarrow$ Q-balls with large charges could be generated!

Do Q-balls thermalize?

Estimate the free energy transferred to a cold Q-ball from the plasma:



$$\frac{dF}{dt} \sim (\text{flux}) \times (\text{no. of interactions}) \times (\text{energy transferred})$$
$$\sim T^2 (\frac{T}{M})^3 Q^{1/4}.$$

Integrate this from T_{in} onwards, with

$$t = \frac{M_0}{T^2}, \quad M_0 \approx 3 \times 10^{17}\,\text{GeV}.$$

Require that $\Delta F \sim F_Q = \text{const.} \times TQ^{3/4}$.

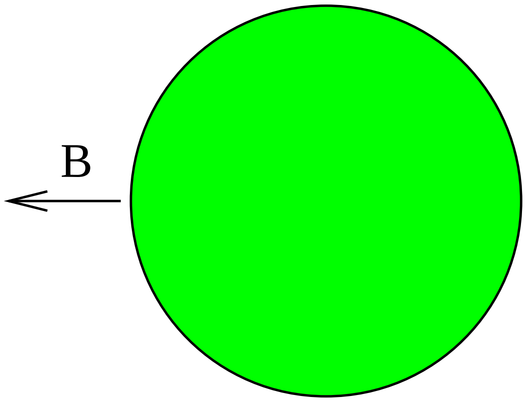
$\Rightarrow$ Q-balls get thermalized if the initial temperature is

$$T_{\text{in}} > M \left(\frac{M}{M_0}\right)^{1/2} Q^{1/4}.$$

For typical values of $M \sim 1 - 10$ TeV and $Q \sim 10^{20} - 10^{30}$, $T_{\text{in}} \gtrsim 1 \ldots 10^4$ GeV, which is true for most inflationary models.

Are Q-balls chemically stable?

Assume that initially, all the charge is in the Q-ball. This is not the equilibrium situation. How much evaporates?



$$\frac{dQ}{dt} \sim (\text{density difference}) \times (\text{surface area}) \times (\text{velocity})$$
$$\sim -\frac{T^2 Q^{1/4}}{M(T)}.$$

Integrating again in an expanding Universe, one gets

$$Q \sim Q_0 \left(1 - \frac{M_0}{M Q_0^{3/4}} \ln\left(\frac{M}{T_{\text{fin}}}\right)\right)^{4/3},$$

where $T_{\text{fin}} \sim$ mass of the lightest SUSY partners.

$\Rightarrow$ Q-balls survive evaporation if the initial charge Q_0 is

$$Q_0 \gtrsim \left(\frac{M_0}{M} \ln\left(\frac{M}{T_{\text{fin}}}\right)\right)^{4/3}.$$

If, for example, $M \sim 1$ TeV, then all Q-balls with $Q_0 > 10^{20}$ stay till the present time. Such values are naturally produced.

Cosmological implications:

Assume that Q-balls contribute to cold dark matter a factor $\Omega_{\text{DM}} < 1$. Then their number density is

$$n_Q^{-1} T^3 \approx 10^6 \frac{M}{m_p} \frac{Q^{3/4}}{\Omega_{\mathrm{DM}}}.$$

On the other hand, the charge evaporated from a Q-ball is

$$\Delta Q \sim \frac{M_0}{M} \ln\left(\frac{M}{T_{\text{fin}}}\right) Q^{1/4}.$$

Thus, the number density of baryons n_B in the plasma is related to the number density of Q-balls n_Q as

$$n_B \sim n_Q \Delta Q.$$

It follows that

$$\eta = \frac{n_B}{n_\gamma} \sim 10^{-10} (M/\mathrm{TeV})^{-2} (Q/10^{26})^{-1/2}.$$

According to primordial nucleosynthesis, $\eta \approx 10^{-10}$.

The correct order of magnitude for $M \sim 1...10$ TeV, $Q \sim 10^{22}...10^{28}$!

Can Q-balls be observed today?

Mass: $M_Q \sim (10^{19}...10^{26})m_p = (10^{-5}...10^{2})$ g.

Radius: $R_Q \sim (100...1000)$ fm.

Average distance: $n_Q^{-1/3} \sim (10^3...10^6)$ km.

Velocity: $v \sim 10^{-3}c$.

Flux on 1 km^2: $(1...10^{-7})$/year.

The experimental signature: Kusenko et al

Inside the Q-ball, there is no confinement. Hence baryons split, and can emerge again as pions, liberating $\sim$ 1 GeV of energy.

The track: 100 GeV/cm in water.

$\Rightarrow$ one cannot miss it, but you have to have a large detector! Constraints from neutrino detectors, monopole searches $\Rightarrow$ not ruled out.

There are also astrophysical constraints from neutron stars. (Q-balls can pile up, eat mass, and the star explodes!)

Conclusions:

* ⋆ Q-balls are charged non-topological solitons, constructed of a scalar field.
* ⋆ Q-balls could exist in supersymmetric theories, if supersymmetry is broken in a suitable way; the role of the charge is then played by the baryon number.
* ⋆ Q-balls can be generated in the Early Universe and can survive till the present day.
* ⋆ Q-balls can contribute simultaneously to dark matter and to baryon asymmetry.

Ultimately, Q-balls should be detected!