The (MSSM) electroweak phase transition

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The physics problem:

The Universe has a matter-antimatter (baryon) asymmetry.

Primordial nucleosynthesis quantifies the asymmetry:

$$\frac{n_B}{n_\gamma} \approx 10^{-10}.$$

The last instance where it could have been generated is the electroweak phase transition ($T \sim 100$ GeV, $t \sim 10^{-11}$ s).

At the same time, this is the possibility requiring the least assumptions beyond established (or soon-to-be) physics, and the only scenario testable at LEP and the Tevatron.

Other motivations for studying the EW phase transition: magnetic fields, inhomogeneities, gravitational radiation.

The scenario of electroweak baryogenesis:

$T \gg T_c$

Inflation dilutes away any original asymmetry?

No B-L violation?

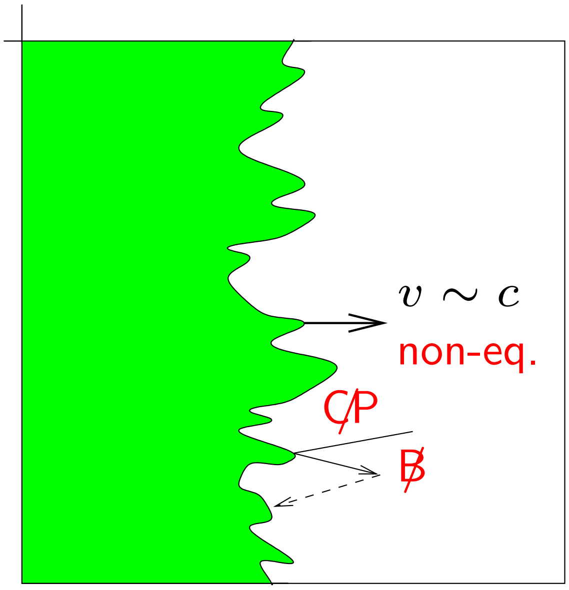
$T \gtrsim T_c,\, v/T \sim 0$

B+L $\xleftrightarrow{\text{anomaly}}$ 0

$\dot{n}_{B+L} \sim \alpha_W^5 \ln(\frac{c}{\alpha_W}) T^4$

B=0

$T \sim T_c$: generate $B \neq 0$



$T \lesssim T_c$: preserve $B \neq 0$

B+L $\not\longleftrightarrow$ 0

$\dot{n}_{B+L} \sim e^{-45(v/T)} T^4$

B$\neq$0

Many parts can be studied with lattice (given m_H)!

Why does one need simulations?
(even though the theory is weakly coupled and perturbation theory works perfectly at $T = 0$)

The IR-problem at finite T:
there is a new scale T, and there are new expansion parameters!

$$\frac{g^2}{4\pi}; \quad \frac{g^2 T}{4\pi m_W}, \qquad m_W \sim \frac{gv}{2};$$

$$\frac{g_S^2 T}{4\pi m_{\tilde{t}_R}}, \qquad m^2_{\tilde{t}_R} \sim -\tilde{m}_U^2 + \# T^2 + \frac{h_t^2 v^2}{2}$$

This comes from:

$$T = 0 \quad \Rightarrow \quad g^2$$

$$T > 0 \quad \Rightarrow \quad g^2 n_b(E) = \frac{g^2}{e^{E/T} - 1} \overset{E \ll T}{\sim} \frac{g^2 T}{E}.$$

In perturbation theory the transition is between $v = 0$, $v \sim T$
$\Rightarrow$ the perturbative description may be unreliable!

Concrete indications of bad convergence:

- 2-loop corrections are large Espinosa

- the gauge and renormalization scale dependence of the 2-loop result (formally of the 3-loop order) are relatively large.

However, 4d simulations are very demanding!

- chiral fermions (the top quark) are important.
- the coupling is weak $\Rightarrow$ it is a multiscale system:

$$a \ll \frac{1}{\pi T} \ll \frac{1}{\sqrt{2} g_S T} \ll \frac{1}{g_S^2 T} \ll \frac{1}{g_W^2 T} \ll Na,$$

so that the lattices need be very large.

But: have been done for the SU(2)+Higgs model! Aoki et al; Fodor et al

Another approach: 3d effective theories!

Combine the best parts of perturbation theory and simulations: integrate out massive modes perturbatively, study light modes non-perturbatively! Allows to

- study the full theory with chiral fermions, for realistic (small) gauge couplings.
- obtain an infinite volume and continuum extrapolation with a relative error $\lesssim 10\%$. The reduction error is also assumed to be $\lesssim 10\%$, so that this is the error of the final result!
- save several orders of magnitude of cpu-time.

Perturbative reduction into 3d

Ginsparg; Appelquist, Pisarski; Nadkarni; Reisz; Kajantie, Rummukainen, Shaposhnikov; Jakovác, Patkós; Braaten, Nieto;. . .

The basic idea: factorization

- integrate out analytically $p \gtrsim \pi T, gT$: non-zero Matsubara modes, in particular all fermions!
  (no IR-problems; expansion in $\frac{g^2}{(4\pi)^2}, \frac{g}{4\pi}$)

- study numerically $p \lesssim g^2 T$
  (soft bosonic modes with IR-problems)

Left: a purely bosonic 3d theory for Matsubara zero-modes.

The formal statement:

Kajantie et al

All static Green's functions at $T \sim T_c$ can be obtained with relative error

$$\frac{\delta G}{G} \sim \mathcal{O}(g^3)$$

from a 3d theory, when its parameters are suitably fixed. Fixing can be done perturbatively, with a systematic procedure.

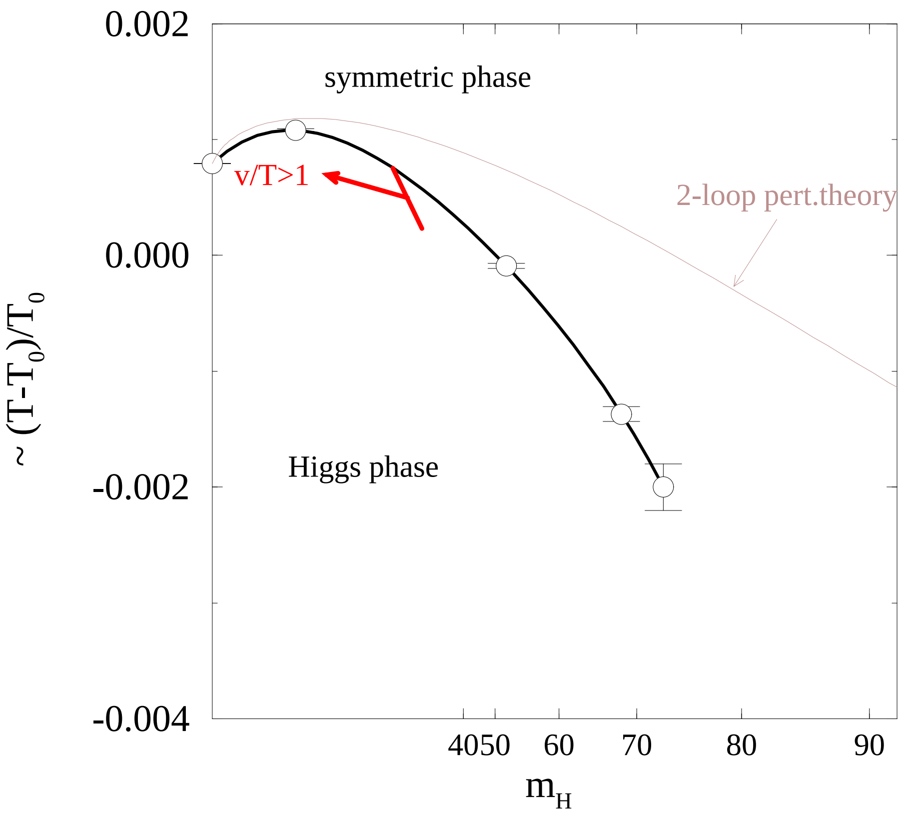
The numerical error involved in the reduction step:

$\lesssim 5\%$ in the Standard Model and $\lesssim 10\%$ in the MSSM. Does not work for the QCD phase transition!

The Standard Model

$$\mathcal{L}_{3\text{d}} = \frac{1}{4}G^a_{ij}G^a_{ij} + \frac{1}{4}F_{ij}F_{ij} + (D_i\phi)^\dagger D_i\phi + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2$$

The phase diagram:



Endpoint:
existence & location:
Kajantie et al; Karsch et al;
Gürtler et al;
universality class:
Rummukainen et al;
Neuhaus et al;
mass spectrum:
Schiller et al;
with 4d simulations:
Aoki et al; Fodor et al;

Z-strings:
Chernodub et al;

Rate observables:
Moore; Smit et al

Conclusions on perturbation theory:

the non-pert. transition is weaker and ends at $m_H \lesssim 80$ GeV.

Conclusions on cosmology:

although in principle all the ingredients for baryogenesis are there already in the SM, in practice one needs something more, to have non-equilibrium! (& more CP-violation)

In the MSSM, in contrast, baryogenesis could work!

1. There can be more CP-violation!
(New phases in the scalar sector.)

Carena et al; Cline et al

2. There can be a stronger transition!
(More light bosonic d.o.f.'s: the squarks.)

Bödeker et al; Carena at al; Cline and Moore; Losada

The regime: right-handed stops are light, $m_{\tilde{t}_R} < m_{\rm top}$, and the Higgs is anything allowed by MSSM, $m_H =$ 90...110 GeV.

The form of the effective theory:

two Higgs doublets H^1, H^2;
the SU(2) and SU(3) gauge fields A^a_μ, C^A_μ;
higgsinos and gauginos; $\Rightarrow$
quarks of the $3^{\rm rd}$ generation q_L, t_R, b_R;
squarks of the $3^{\rm rd}$ generation Q, U, D.

$$
\begin{aligned}
\mathcal{L}_{\rm 3d} &= \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}G^A_{ij}G^A_{ij} + \gamma_3 H^\dagger H U^\dagger U \\
&+ (D^w_i H)^\dagger (D^w_i H) + m^2_{H3} H^\dagger H + \lambda_{H3}(H^\dagger H)^2 \\
&+ (D^s_i U)^\dagger (D^s_i U) + m^2_{U3} U^\dagger U + \lambda_{U3}(U^\dagger U)^2 .
\end{aligned}
$$

Discretization in the standard way.

$$\begin{aligned}\mathcal{L}_{\rm latt} &= \beta_G^{\rm SU(3)} \sum_{i<j}\left[1-\frac{1}{6}\,{\rm Tr}\left(P_{ij}^S + P_{ji}^S\right)\right] \\ &+ \beta_G^{\rm SU(2)} \sum_{i<j}\left[1-\frac{1}{2}\,{\rm Tr}\,P_{ij}^W\right] + \ldots\end{aligned}$$

Renormalization: perturbation theory gives an exact result at 2-loop level. Moreover, an $\mathcal{O}(a)$ improvement is known for most quantities. Moore

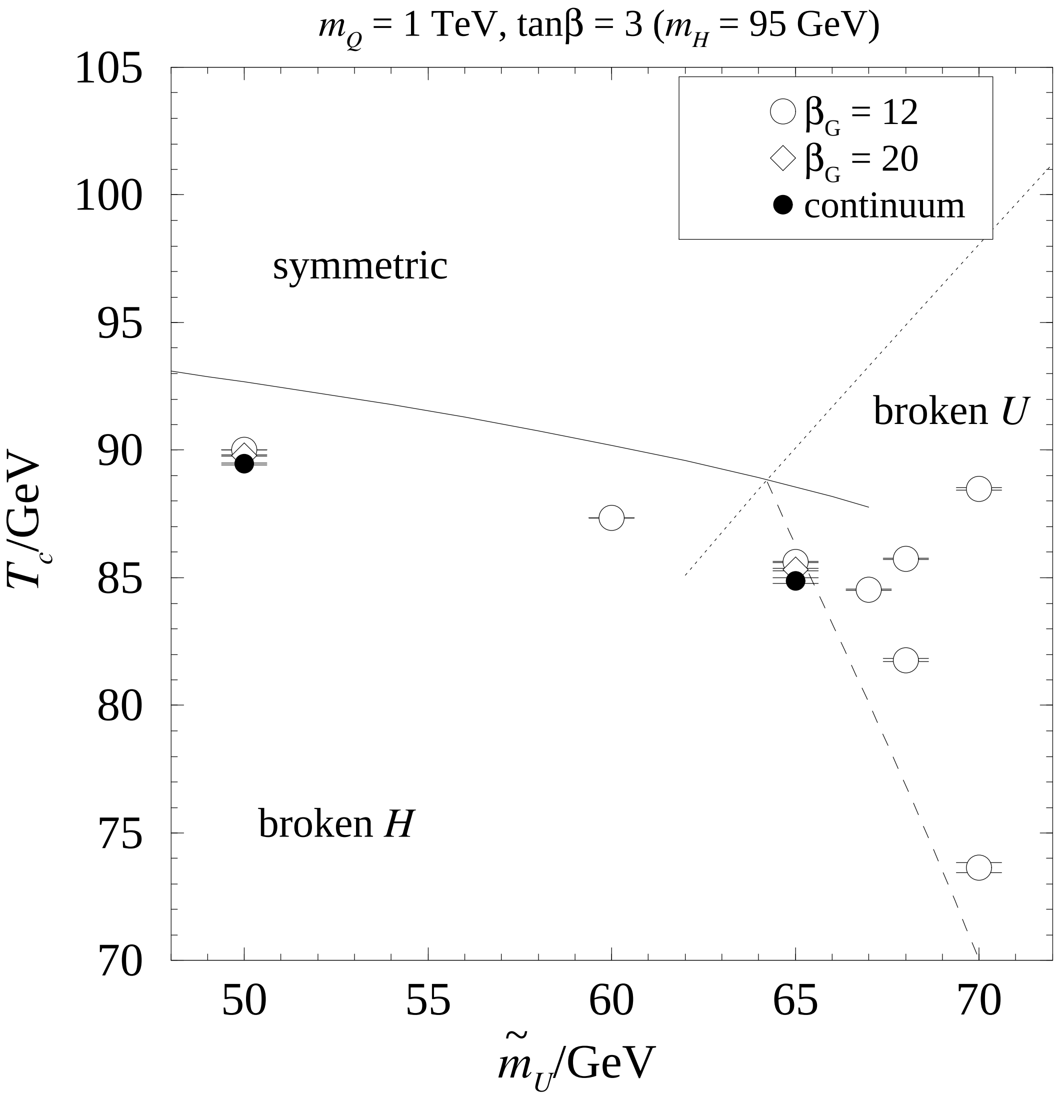
Infinite volume and continuum extrapolations:

$\tilde{m}_U$	$\beta_G^{\rm SU(3)}$	volumes		
50	12	12^3 16^3 20^3_m		
		$12^2 \times 36_m$	$16^2 \times 48_m$	
		$20^2 \times 72_m$	$24^2 \times 128_m$	
	20	$16^2 \times 64_m$	$20^2 \times 72_m$	$24^2 \times 72_m$
		$28^2 \times 80_m$	$32^2 \times 128_m$	

Simulations with a parallel machine Cray T3E, 64 processors. The total cpu-usage is $\sim 2 \times 10^{16}$ flop.

Measured: $\langle H^\dagger H\rangle$, $\langle U^\dagger U\rangle$, etc $\Rightarrow$ latent heat, surface tension, critical temperatures, correlation lengths.

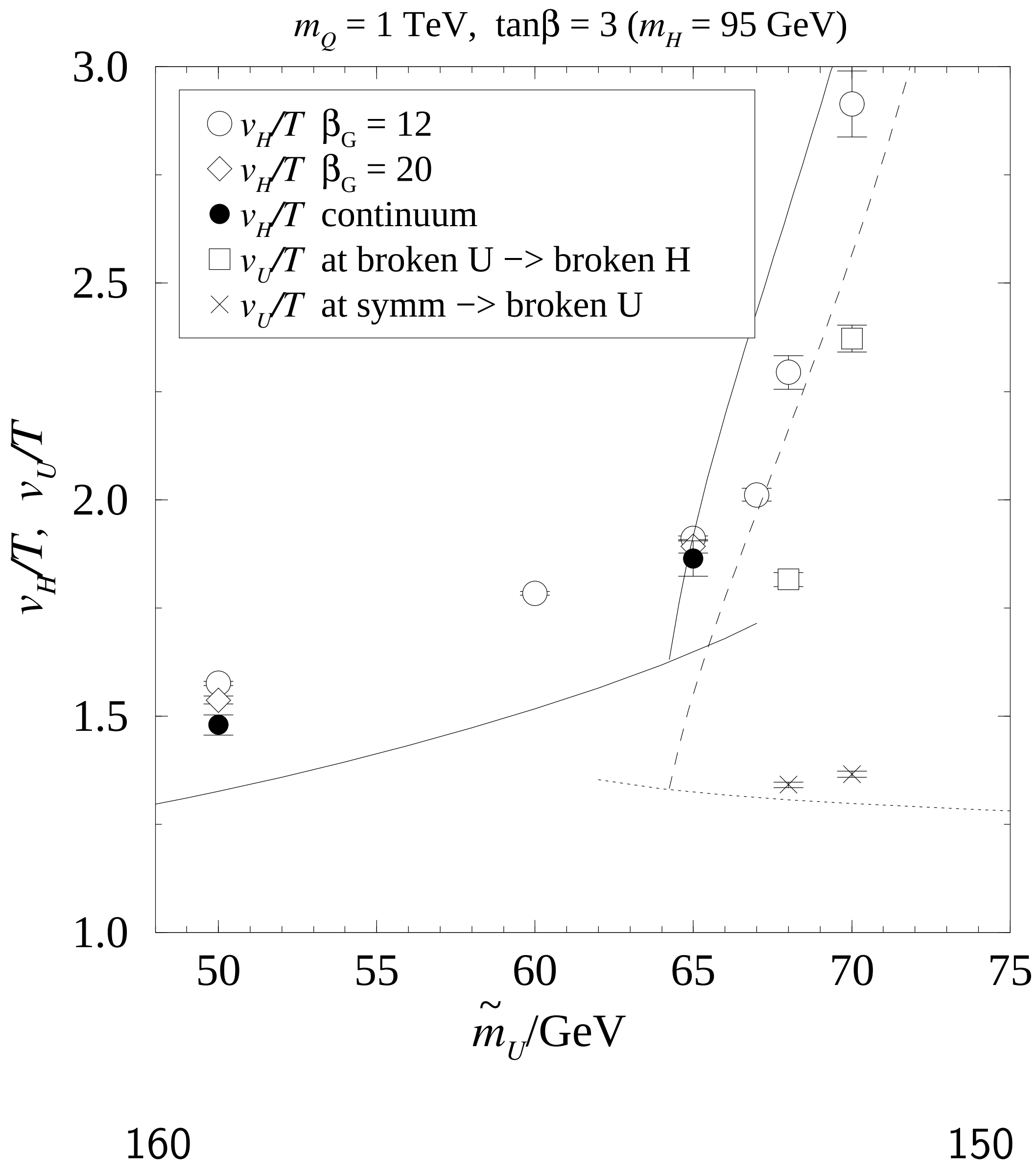
The phase structure:



160 150

$$m_{\tilde{t}_R} = \sqrt{m_{\rm top}^2 - \tilde{m}_U^2}$$

The sphaleron rate in the broken phase (the scalar vev, $v_H^2/2 = \langle H^\dagger H \rangle$):



$$m_{\tilde{t}_R} = \sqrt{m_{\rm top}^2 - \tilde{m}_U^2}$$

Conclusions on perturbation theory:

⋆ The critical temperatures are a bit smaller than in 2-loop perturbation theory, but the phase structure is the same.

⋆ The transition is stronger than in 2-loop perturbation theory! However, the difference is not terribly large.

Conclusions on cosmology:

⋆ Higgs masses up to 105...110 GeV are allowed for baryogenesis (for $m_{\tilde{t}_R} < m_{\rm top}$).

⋆ There is, in principle, the possibility of a two-stage transition with an exotic intermediate phase.

Future directions:

⋆ other parameter values (larger Higgs masses, non-vanishing squark mixing); more statistics; more observables; . . .

⋆ real time rate observables (the sphaleron rate, . . .)

⋆ 4d simulations and the accuracy of dimensional reduction

⋆ . . .

⋆ two Higgs doublets and CP-violation!

Two Higgs doublets:

$$
\begin{aligned}
\mathcal{L}_{\rm 3d} &= \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}G^A_{ij}G^A_{ij} \\
&+ (D^s_iU)^\dagger(D^s_iU) + m^2_U U^\dagger U + \lambda_U (U^\dagger U)^2 \\
&+ U^\dagger U[\gamma_1 H_1^\dagger H_1 + \gamma_2 \tilde{H}_2^\dagger \tilde{H}_2 + (\gamma_3 H_1^\dagger \tilde{H}_2 + \text{H.c.})] \\
&+ (D^w_i H_1)^\dagger (D^w_i H_1) + (D^w_i \tilde{H}_2)^\dagger (D^w_i \tilde{H}_2) \\
&+ m^2_{H1} H_1^\dagger H_1 + m^2_{H2} \tilde{H}_2^\dagger \tilde{H}_2 + (m^2_{H12} H_1^\dagger \tilde{H}_2 + \text{H.c.}) \\
&+ \lambda_{H1}(H_1^\dagger H_1)^2 + \lambda_{H2}(\tilde{H}_2^\dagger \tilde{H}_2)^2 \\
&+ \lambda_{H3} H_1^\dagger H_1 \tilde{H}_2^\dagger \tilde{H}_2 + \lambda_{H4} H_1^\dagger \tilde{H}_2 \tilde{H}_2^\dagger H_1 \\
&+ (\lambda_{H5} H_1^\dagger H_1 H_1^\dagger \tilde{H}_2 + \lambda_{H6} \tilde{H}_2^\dagger \tilde{H}_2 H_1^\dagger \tilde{H}_2 + \text{H.c.}) \\
&+ (\lambda_{H7}(H_1^\dagger \tilde{H}_2)^2 + \text{H.c.}).
\end{aligned}
$$

The parameters $\gamma_3, m^2_{H12}, \lambda_{H5}, \lambda_{H6}, \lambda_{H7}$ can be complex: e.g.,

$$
\lambda_{H7} = \frac{3}{32\pi^2}\left[2h_t^4\left(\frac{A_t^*\mu^*}{m_Q^2}\right)^2 \ln\frac{4\pi e^{1-\gamma}T}{m_Q} + \frac{31\zeta(5)}{64}\left(\frac{M_2\mu^*}{(\pi T)^2}\right)^2\right],
$$

where A_T, μ are trilinear couplings in the squark mixing matrix.

CP-violating operators exist in this theory ($\text{Im}\, H_1^\dagger \tilde{H}_2$) and can be studied non-perturbatively!