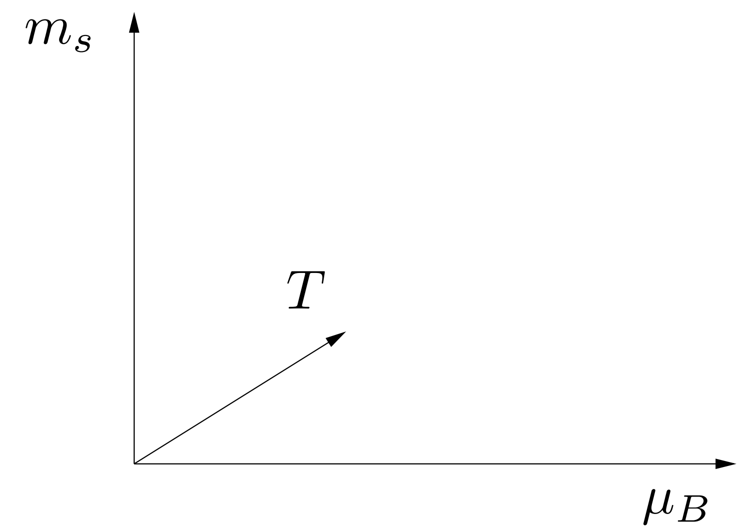
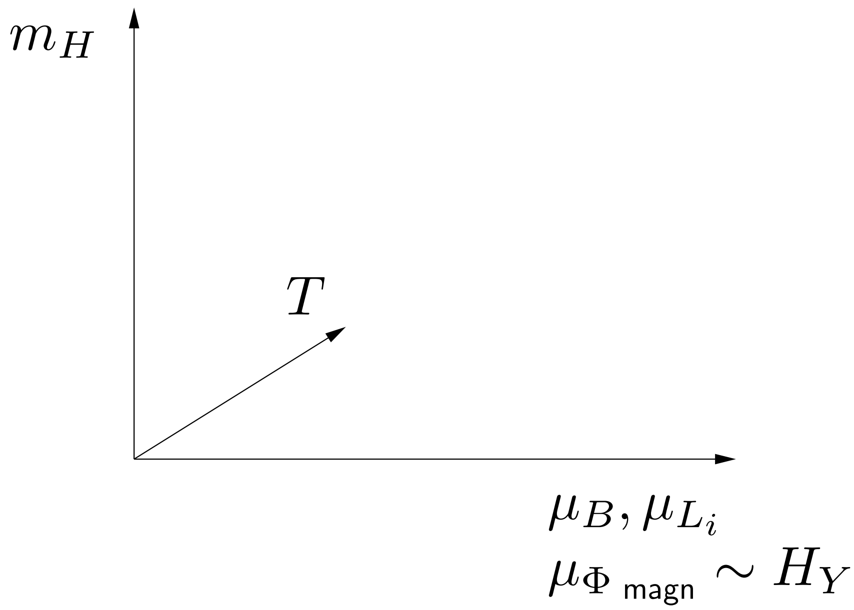
QCD:



SU(2)×U(1) EW:



- Why are we interested?
- Methods
- Results
- Connections to QCD

Physics motivation

- (m_H, T): electroweak baryogenesis in cosmology, at the lowest possible energy scale.

Status: Does not work for the Standard Model, because there is no 1st order phase transition for $m_H > 72$ GeV.

Could work in the MSSM if $95 \lesssim m_H \lesssim 105$ GeV and $m_{\tilde{t}_R} < m_{\,\text{top}}$. This regime is being studied at LEP and the Tevatron!

- (m_H, T, H_Y): external magnetic fields are observed in the galaxies and could affect baryogenesis.

Status: Cannot probably save electroweak baryogenesis in the Standard Model, but other implications remain intriguing.

- $(m_H, T, \mu_B, \mu_{L_i})$: $\mu_B/T \sim 10^{-10}$ is the number to be explained. However, from the point of view of the phase diagram, so small numbers do not have practical significance.

The phase diagram for $(m_H, T, H_Y = 0)$

The symmetry breaking "order parameter", the Higgs field, and the fact that the symmetry is "restored" at high temperatures, can be identified already in perturbation theory. However, all the details of the transition are non-perturbative.

The problem: there are many different scales in the problem,

$$m \sim \pi T, gT, g^2 T,$$

and the expansion parameter at finite temperature is $\sim g^2 T/m$.

Moreover, direct 4d lattice simulations are extremely demanding for a system with multiple length scales. Besides, chiral fermions such as the top quark cannot be put on the lattice at all.

Method: perturbation theory does work for the modes with $m \sim \pi T, gT$. Integrate them out perturbatively and construct an effective theory for the modes with $m \sim g^2 T$!

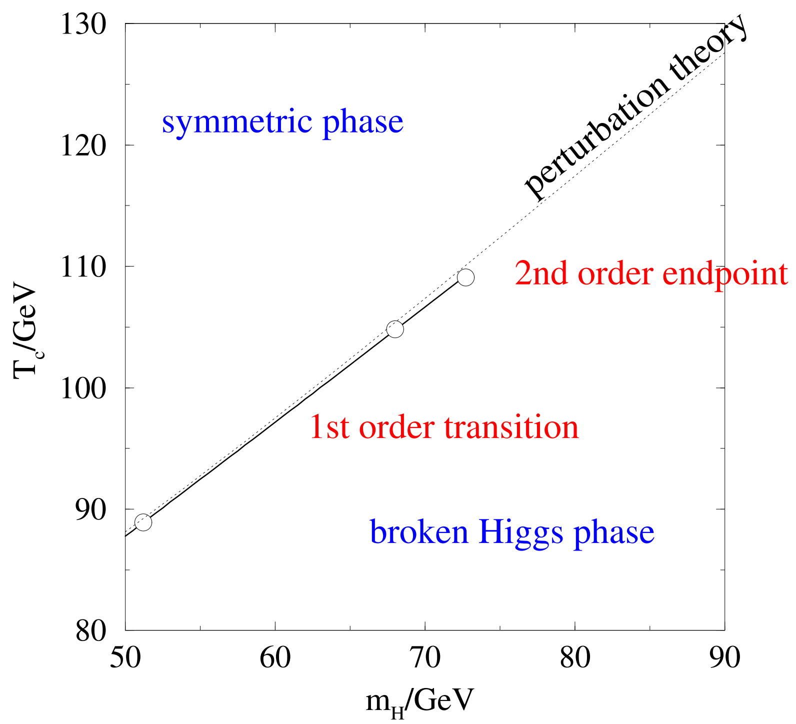
The effective theory is a purely bosonic 3d field theory

$$\mathcal{L}_{3\mathrm{d}} = \frac{1}{4} G^a_{ij} G^a_{ij} + \frac{1}{4} F_{ij} F_{ij} + (D_i \phi)^\dagger D_i \phi + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2,$$

which can be studied with lattice simulations.

The parameters have been computed up to 1-loop and 2-loop levels. The effects of neglected higher-dimensional operators can be estimated systematically, and are $\lesssim \mathcal{O}(5\%)$.

The phase diagram for the Standard Model:



1st order regime:
Kajantie et al;
Ilgenfritz et al;
Csikor et al; Aoki et al;...

Endpoint:
existence & location:
Kajantie et al; Karsch et al;
Gürtler et al; Aoki et al;
Csikor et al;
universality class:
Rummukainen et al

Similar studies have been carried out also for the MSSM.

How does it work for QCD?

[Of course, in QCD even 4d lattice simulations are OK in principle, they are just quite hard for light fermions.]

The effective theory:

Ginsparg; Appelquist, Pisarski; Braaten, Nieto; Kajantie et al;. . .

$$\mathcal{L}_{\text{3d}} = \frac{1}{4}F^a_{ij}F^a_{ij} + \mathrm{Tr}[D_i, A_0][D_i, A_0] + m_D^2 \,\mathrm{Tr}\, A_0^2 + \lambda_A(\mathrm{Tr}\, A_0^2)^2.$$

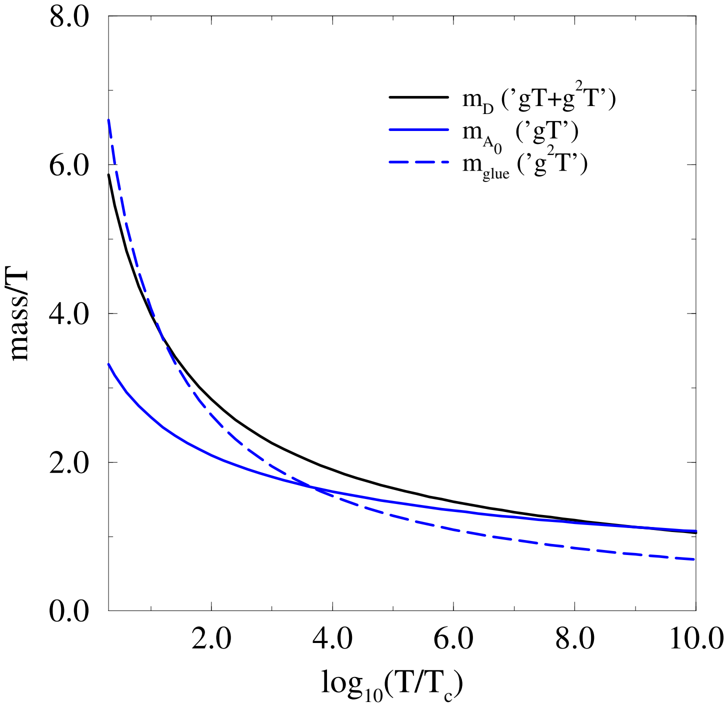
Is it OK down to $T \sim T_c$?

1. The expansion for the parameters converges down to $\sim T_c$, since the scale in α_s is $\sim 2\pi T$.

2. The effects of the higher order operators are governed by the ratios of the mass scales $\sim g^2T, gT$ kept in the effective theory to those $\sim 2\pi T$ integrated out.

The mass scales from the 3d theory:

Kajantie et al;
Laine and Philipsen



Most mass scales within the effective theory are $\sim 2\pi T$ already at around $2T_c \Rightarrow$ higher order operators are important.

Other ways to see the problem at $T \sim T_c$:
For pure SU(N), the transition is driven by the Z(N) symmetry. This has been lost in the effective theory, since the relevant field configurations have $gA_0 \sim 2\pi T$.

With fermions, even the order of the phase transition depends on the fermionic content, while in the effective theory the fermions only appear in the parameters.

The phase diagram for (m_H, T, H_Y):

A fixed magnetic flux $\Phi_{\text{magn}} \sim Q$:

$$\rho_{\text{microcan.}} \quad = \quad \frac{1}{Z} \exp\left(-\frac{H}{T}\right) \delta\left(\int d\vec{s} \cdot \vec{B}_Y - \Phi_{\text{magn}}\right).$$

A fixed field strength $H_Y \sim \mu$:

$$\rho_{\text{can.}} = \frac{1}{Z} \exp\left[-\frac{1}{T}(H - \int d^3x\, \vec{H}_Y \cdot \vec{B}_Y)\right].$$

• At finite T, the effective theory is just the same as earlier on.

• Simulations are easiest with the microcanonical ensemble: just change the boundary conditions for the field A_i:

$$\Phi_{\text{magn.}} = \int d\vec{s} \cdot \vec{B}_Y = \int dx_1 dx_2 F_{12} = \oint ds_i A_i.$$

This is somewhat analogous to simulations of quenched QCD with a finite baryon number.

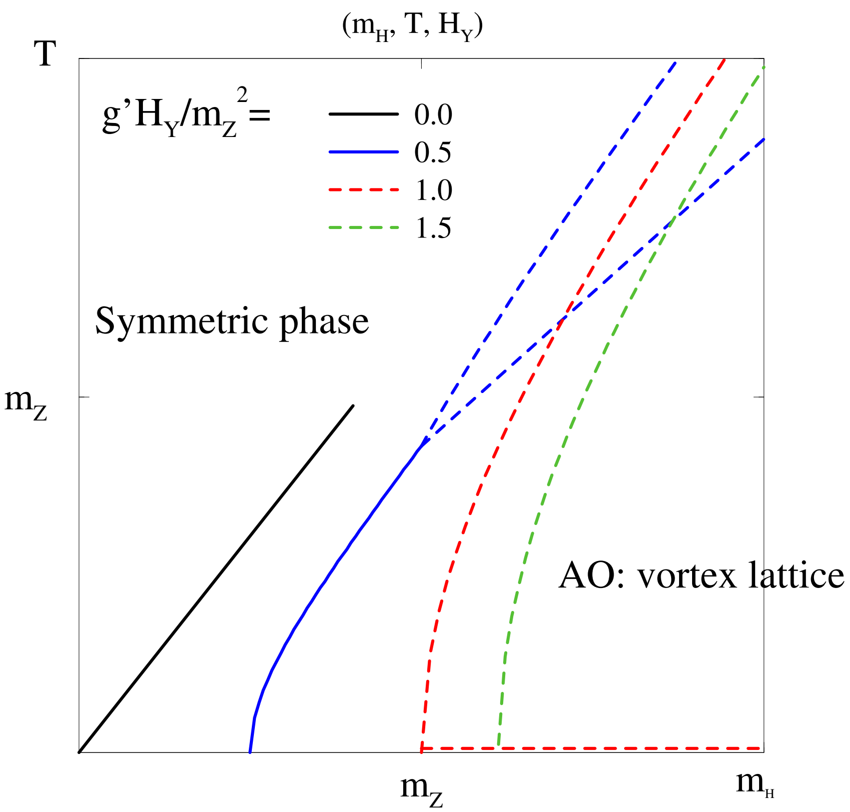
Engels, Kaczmarek, Karsch, Laermann

Physics: Landau-levels. E.g., in the broken phase,

$$E^2_{W^\pm} \sim m^2_W + g' H_Y \cos^2\theta_W (-1 + 2n), \quad n = 1, 2, ...$$

There is a regime where the lowest E^2's are negative in both phases $\Rightarrow$ instability, the Ambjørn-Olesen phase (vortex lattice).

The result for the phase diagram:



Conclusions

Similarly to QCD, the electroweak sector of the Standard Model has a non-trivial phase diagram, both at zero and non-zero T.

The main properties (order, latent heat, ...) of the phase transitions are non-perturbative. Nevertheless, the lightest degrees of freedom can be easily identified, and the weak coupling allows to systematically construct effective field theories, which can then be studied with lattice.

The results are of importance for cosmological issues such as baryogenesis (but have also some analogies in QCD).