Finite temperature phase transitions in cosmology

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Particle content of the Standard Model (SM):

QCD, SU(3):
quarks, gluons —
appear as π's (not stable), p, n, elements.
Typical energy scale: $E \sim m_\pi \sim 100$ MeV.

EW, SU(2)×U(1):
quarks, leptons $(e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau)$, $W^\pm, Z^0, \gamma$, the Higgs —
appear as e, ν's, γ (others not stable).
Typical energy scale: $E \sim m_Z \sim 100$ GeV.

The macroscopic properties of an ensemble of SM particles can be described by its global **conserved numbers**. The only are baryon number B ($= \frac{1}{3}$ for quarks) and lepton numbers L_i ($= 1$ for e, ν_e, ...). Except that one combination, $B + \sum_i L_i$, is not conserved because of an electroweak anomaly!

(The electric charge is locally conserved too, but any homogeneous system as a whole must be neutral.)

What is the present Universe composed of? (in terms of e)

Massless gauge fields

Electromagnetic radiation & classical magnetic fields: $\mathcal{O}(\%)$

Matter — cannot decay because of conserved numbers!

Baryon number B in light elements H, D, He, Li, ...: $\sim 5\%$.

Lepton numbers L_i can be in a ν background (not observed).

There is also non-baryonic "Dark Matter": $\sim 25\%$.
Related perhaps to supersymmetric R-parity, or L_i?

Gravity

Gravitational radiation & cosmological constant: $\sim 70\%$.

Another relic is a spatial fluctuation in any of these!

Can we understand how these numbers come about?

1. What is the underlying particle physics model?
2. How to reliably do statistical physics in an expanding background with it?

What would the Universe look like in complete **equilibrium**?

By definition, equilibrium means a system without any time dependence, characterized only by such thermodynamic variables as T, B, L_i, or T, μ_B, μ_{L_i}.

In practice, the system is in equilibrium in an adiabatic sense: there is an inherent dynamical time scale τ, and an external observation time scale t: equilibrium is obtained for $t \gg \tau$, when dynamical processes die out.

For instance, on a semiclassical Boltzmann equation level,

$$\frac{df(x,p,t)}{dt} = -\frac{1}{\tau}[f(x,p,t) - f_0(T,\mu_i)].$$

For $t \gg \tau$ exponentially close to f_0 for any initial condition.

If there were complete equilibrium today, cosmological relics $f_0^{(j)}(T, \mu_i)$, $j =$ different particles, would look very different from what is observed!

For instance, the cosmic microwave background (CMB) would be absent! The present Universe is really cold ($T = 0.0$ K), and CMB was predicted (Gamow 46) and observed (Penzias, Wilson 65) to be a non-equilibrium remnant of an earlier epoch!

Similarly, there would be no light elements: all B would reside in iron, which has the largest binding energy.

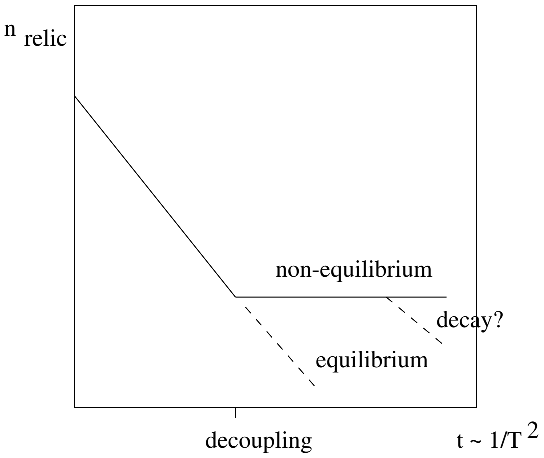
In fact, there would not even be any net baryon number! After inflation all net charges (B, L_i) vanish. If there is full equilibrium after that, nothing can be generated.

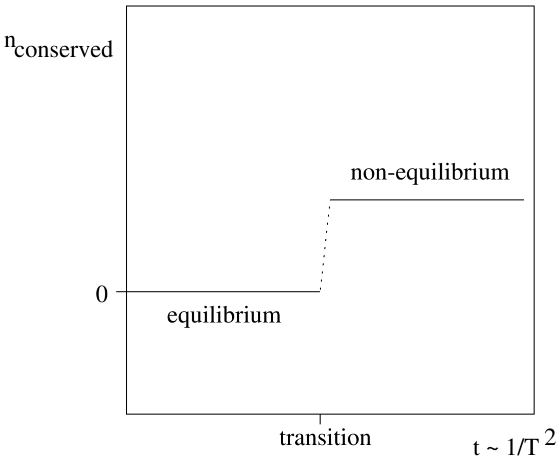
So we necessarily need non-equilibrium!

There are two basic ways of obtaining non-equilibrium:

1. The time t is the age of the Universe. If $\tau > t$, microscopic reactions are too slow to keep up with the expansion, and different particle species "decouple", or "freeze out". This is the simplest way, and the standard explanation for all observed (non-conserved) relics.

For instance, in CMB, the plasma neutralises at $T \approx 3000$ K, the interaction rate of photons falls down ($\tau \to \infty$), and the existing blackbody radiation stays.





2. There is also another way: τ can become large because of a phase transition! In a first order phase transition, there is a discontinuity in the equation of state, and in τ. It can be that $\tau \ll t$ before the transition, $\tau \gg t$ after the transition. During the transition, the system supercools, bubbles nucleate and expand, etc. This more non-trivial alternative can in principle be used in a remarkable way!

To quantify when there is equilibrium, recall basics of expansion. Take homogeneous and isotropic metric $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$, ideal fluid $T_\mu^{\ \nu} = \text{diag}\ (e, -p, -p, -p)$, and put into Einstein equations $G_\mu^{\ \nu} = 8\pi G T_\mu^{\ \nu}$:

$$\begin{aligned} \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} &= \frac{8\pi G}{3}e, \\ d(ea^3) &= -pd(a^3). \end{aligned}$$

Here e = energy density, p = pressure, $k = 0, \pm 1$. Observationally, $k = 0$, so assume that for now. We denote

$$G \equiv \frac{1}{m_{\text{Pl}}^2}, \quad m_{\text{Pl}} = 1.2 \times 10^{19}\text{GeV}.$$

Using $s = dp/dT$, $e = Ts - p$ (for $\mu = 0$), eliminate $a(t)$ for

$$\frac{dT}{dt} = -\frac{\sqrt{24\pi}}{m_{\text{Pl}}} \frac{\sqrt{e(T)}}{d\ln s(T)/dT}.$$

For ideal gas $e = (\pi^2/30)g_* T^4$, $g_* = \sum_{m_b \ll T} + \frac{7}{8}\sum_{m_f \ll T} \Rightarrow$

$$t = \frac{3}{4}\sqrt{\frac{5}{\pi^3 g_*}}\frac{m_{\text{Pl}}}{T^2}; \qquad \frac{t}{\text{sec}} = (\frac{\text{MeV}}{T})^2.$$

This gives the cosmological time scale as a function of T.

Compare this with τ for different particle physics processes:
$\tau = \frac{1}{n\sigma v}$, $n =$density, $\sigma =$cross section, $v =$velocity.
Equilibrium if $\tau \ll t \sim m_{\mathsf{Pl}}/T^2$.

photons: relevant processes $e + p \leftrightarrow H + \gamma$, $e + \gamma \leftrightarrow e + \gamma$.
When $T \gg$ eV, n_e is large and there is equilibrium.

light elements: $e + p \leftrightarrow n + \nu$, $p + n \leftrightarrow d + \gamma$, etc. When $T \gg m_n - m_p \sim$ MeV, n_n is large and there is equilibrium.

ν's: $n \sim T^3, \sigma \sim \alpha_w^2 T^2/m_Z^4$: eq. at $T \gg (\frac{m_Z^4}{\alpha_w^2 m_{\mathsf{Pl}}})^{1/3} \sim$ MeV.

BUT:

QCD: $n \sim T^3, \sigma \sim \alpha_s^2/T^2$, eq. at $T \ll \alpha_s^2 m_{\mathsf{Pl}} \sim 10^{15}$ GeV.

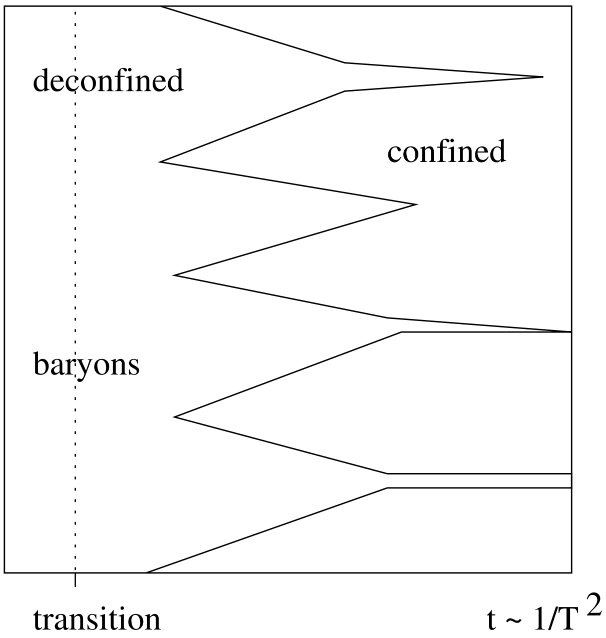
EW: $n \sim T^3, \sigma \sim \alpha_w^2/T^2$, eq. at $T \ll \alpha_w^2 m_{\mathsf{Pl}} \sim 10^{15}$ GeV.

$B + \sum_i L_i$ violation: $1/\tau \sim \alpha_w^5 T \exp(-45|\phi|/T)$,
eq. at $|\phi| \lesssim T$ and $T \ll \alpha_w^5 m_{\mathsf{Pl}} \sim 10^{10}$ GeV.

So “pure” relativistic EW and QCD physics can only fall out of equilibrium via a phase transition! Concentrate now on these.

Cosmological QCD and EW phase transitions could have existed, but did they leave any relics?

The problem with QCD is that there is no obvious interaction rate $1/\tau$ which would switch off across the transition. Mainly the transition can leave a specific spectrum of baryon density fluctuation, but at a small scale. These fluctuations could have effects on nucleosynthesis. Kurki-Suonio
In addition, if by some means magnetic fields were generated, they could be magnified later on by magnetohydrodynamics effects to galactic scale. Son



People are also hopeful that some relics could remain in heavy ion collisions, where expansion rate is $m_{\,\text{Pl}}/m_\pi = 10^{20}$ faster, and it is much easier to switch out of equilibrium.

On the contrary, the EW transition could have left an outstanding relic, the baryon (matter–antimatter) asymmetry!

Kuzmin, Rubakov, Shaposhnikov 1985

The existence of an asymmetry is an experimental fact.

$\star$ no antinuclei are observed in cosmic radiation.

$\star$ there is no observation of significant 150 MeV gamma-radiation, originating from $p\bar{p} \to 5...6\pi \to 12\gamma$ annihilation.

$\star$ nucleosynthesis (the production of ^{4}He, D, Li) $\Rightarrow$

$$\frac{n_B}{n_\gamma} = (1...9) \times 10^{-10}.$$

Nevertheless, we expect that after inflation $B = 0$.

The electroweak phase transition is in a sense the most conservative ($\equiv$ lowest energy, latest) scenario for baryon asymmetry generation, and at the same time the only one testable at LEP and the Tevatron.

How to generate a baryon number $B = \sum_{i=\text{ quarks}} \int d^3x \bar{\psi}_i \gamma_0 \psi_i$?

1. B is violated by the anomaly. The rate is fast at high temperatures, exponentially suppressed at low temperatures.

2. In addition, need non-equilibrium. Otherwise,

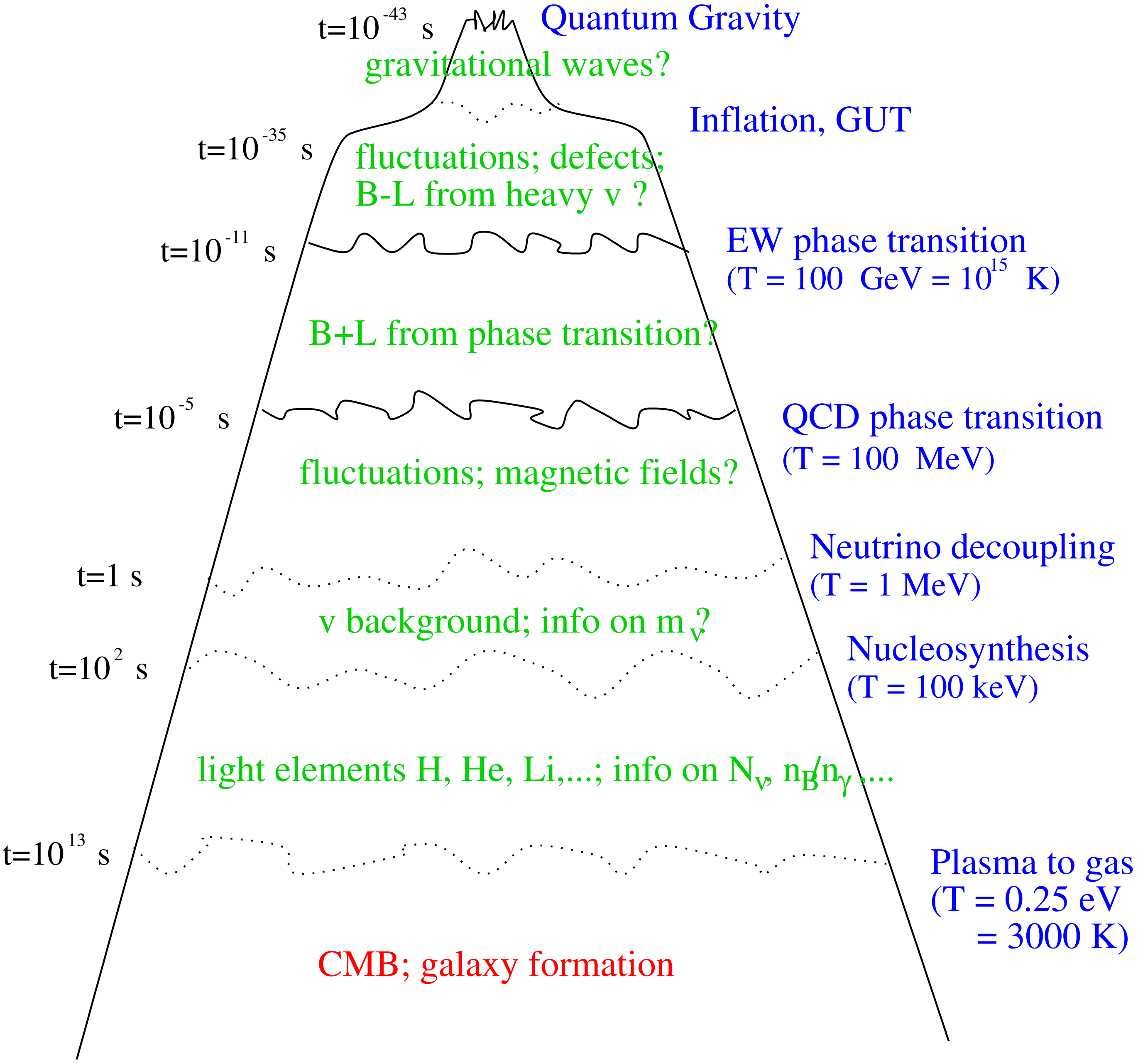
$$\langle B(t) \rangle_T \propto \mathrm{Tr}\, e^{-H/T} e^{iHt/\hbar} B e^{-iHt/\hbar} = \langle B(0) \rangle_T = 0.$$

3. In addition, need CP-violation: B is $P+, C-$.

To generate something out of nothing, one needs some sort of a singularity. It can be provided by a 1st order phase transition. After it, immediately non-equilibrium (if $\Delta|\phi|/T \gtrsim 1$).

The actual generation is a complicated physical process, taking place in the non-equilibrium environment of expanding bubble walls, nucleated after some supercooling.

Summary of general pattern:



Finite temperature field theory

Chemical potentials vanish: $\mu_B/T \propto n_B/n_\gamma \sim 10^{-10}$.

We want to compute:

Free energy: $F(T,V) = -T \ln \mathrm{Tr}\, e^{-\beta H}, \quad \beta = T^{-1}$.

Pressure: $p(T) = -\frac{\partial F(T,V)}{\partial V} = -\frac{F}{V}$.

Entropy density: $s(T) = \frac{dp}{dT}$.

Energy density: $e(T) = Ts(T) - p(T)$.

Expectation values: $\langle \phi \rangle(T) = Z^{-1}\, \mathrm{Tr}\, \phi e^{-\beta H}$, $Z = e^{-\beta F}$.

For all such quantities one can write down a Euclidian path integral expression. For instance,

$$F(T,V) \;=\; -T \ln \int_{\text{b.c.}} \mathcal{D}[A_\mu^a, \phi, \bar{\psi}, \psi] e^{-\int_0^\beta d\tau \int d^3x \mathcal{L}[A_\mu^a, \phi, \bar{\psi}, \psi]},$$

and b.c. over τ are periodic (A_μ^a, ϕ) or anti-periodic ($\bar{\psi}, \psi$).

Phase transition, in general, is a non-analyticity in $p(T)$.
First order transition has a discontinuity in dp/dT, or e.

Classical condensed matter transitions are often described by some **order parameter** ϕ (e.g., magnetisation). The system has a global **symmetry**, like Z(2) or O(N). The transition is between a symmetric phase where $\langle\phi\rangle = 0$, and a phase where the symmetry is spontaneously broken, $\langle\phi\rangle > 0$.

In theories with gauge symmetries the situation is different. The order parameter should be gauge invariant, but quantities like $\phi^\dagger\phi$ can never be zero.

Physical phases of gauge theories can then be:
• confinement or Higgs phase, which cannot be differentiated by any order parameter, but in the latter perturbation theory usually works, in the former not.
• Coulomb phase, if there is an abelian U(1) gauge group. Then there is an effective order parameter, the photon mass, and the symmetry can be broken, like in superconductivity.

If there is no symmetry, there **need not** be any transition!
A classic example is the phase diagram of water and vapour. We call a transition without any non-analytic points a **crossover**.

The properties of phase transitions are non-perturbative, even in a weakly coupled theory! This is the so called "infrared problem" of finite temperature field theory.

Linde 1980;
Gross, Pisarski, Yaffe 1981

The reason: it is a multiscale system. There are thermal fluctuations at the momentum scale $p \sim 2\pi T$, but also soft collective modes at $p \sim gT, g^2T$, generated by interactions.

Expansion parameters related to different scales are different:

$$\epsilon_b \sim \frac{1}{\pi} g^2 n_b(p) = \frac{g^2}{\pi(e^{p/T} - 1)} \overset{p<T}{\sim} \frac{g^2 T}{\pi p}.$$

Then $p \sim 2\pi T$ is still OK, but the other expansion parameters are large, and give odd powers, or no powers at all.

For fermions, the problem is less severe at weak coupling:

$$\epsilon_f \sim \frac{1}{\pi} g^2 n_f(p) = \frac{g^2}{\pi(e^{p/T} + 1)} \lesssim \frac{g^2}{\pi}.$$

When is the coupling weak? QCD vs EW.

Why is $2 \gg 1$?? [K. Kajantie]

The difference is in the nature of the transition.

$$\Lambda_{\overline{\rm MS}} = Q \exp(-\frac{48\pi^2}{22N_c - 4N_f - N_s} \frac{1}{g^2(Q)}).$$

QCD: $g_s(1 \text{ GeV}) = 2 \Rightarrow \Lambda_{\overline{\rm MS}}^{-1} = 1$ fm.
On the other hand, $T_c \sim \Lambda_{\overline{\rm MS}}$,
correlation lengths $\sim 1/\pi T, 1/g_s T, 1/g_s^2 T \approx \Lambda_{\overline{\rm MS}}$.

EW: $g_w(m_Z) = \frac{2}{3} \Rightarrow \Lambda_{\overline{\rm MS}}^{-1} = 10^{15} m_Z^{-1}$.
On the other hand, $T_c \sim m_H/g_w$,
correlation lengths $\sim 1/\pi T, 1/g_w T, 1/g_w^2 T \ll \Lambda_{\overline{\rm MS}}^{-1}$.
Zero temperature "confining" effects are not seen.

QCD phase transition

$$\mathcal{L}_{\text{QCD}} = \frac{1}{2}\mathrm{Tr}\, F_{\mu\nu}F_{\mu\nu} + \sum_i \bar{\psi}_i[\gamma_\mu D_\mu + m_i]\psi_i,$$

The low temperature phase is "confining" (there are no massless excitations even for $m_i \to 0$), and the chiral symmetry relating different fermion flavours is broken ($\langle\bar{\psi}\psi\rangle > 0$). However, in the general case with finite and non-degenerate m_i, chiral symmetry is broken explicitly, and there is no order parameter related to confinement.

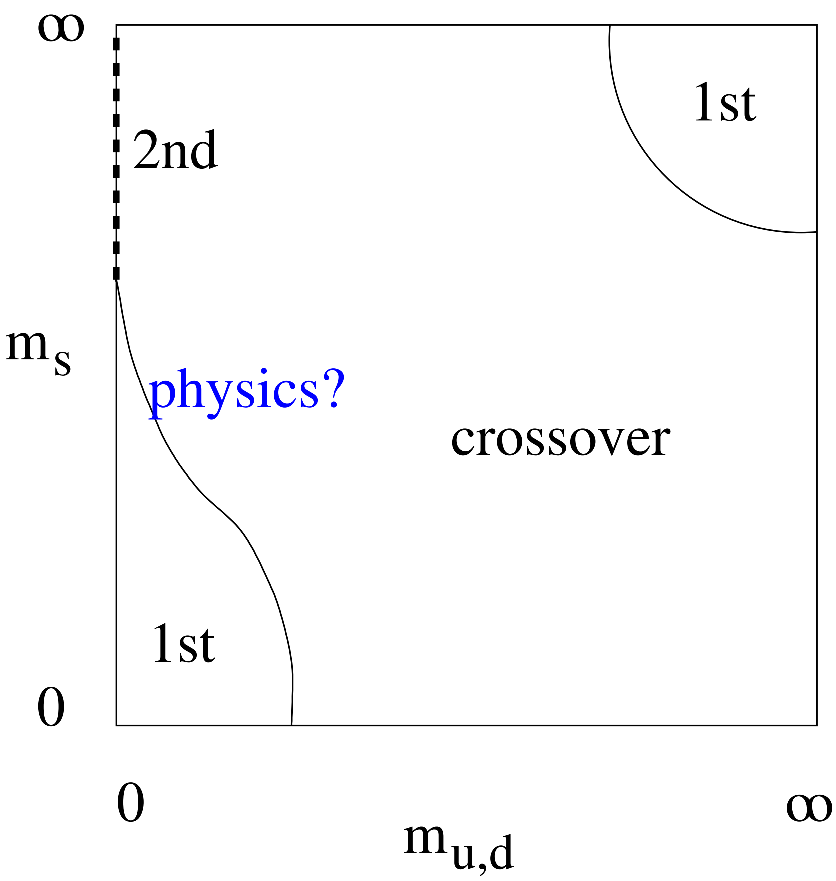
On the other hand, we expect an almost free quark–gluon plasma at high T, because of asymptotic freedom.

So there may or may not be a transition between the two, and to find out requires a completely non-perturbative analysis.

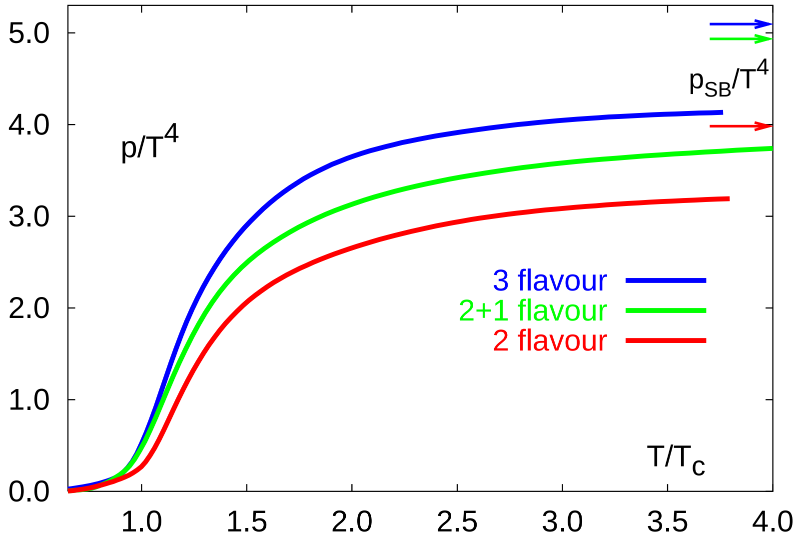
In special cases, though, there are symmetries, and then one can also propose effective models based on symmetries alone. Their parameters are not computable, but they can still give indications on the order of the transition.

For $N_f = 0$ ($m_i = \infty$), there is a Z(N_c) symmetry driving the transition. And three-state spin (Potts) model in 3d has a 1st order transition. For $N_f > 0$ and equal m_i, there is a chiral symmetry driving the transition. $N_f = 2$ corresponds to O(4), which has a 2nd order transition. $N_f = 3$ turns out to correspond to a spin model with a 1st order transition.

Phase diagram [Kanaya, hep-lat/9804006]:



The real case is an outstanding challenge for lattice simulations. At least the behaviour outside of the critical temperature is known to some extent [Karsch et al, hep-lat/0010027]:

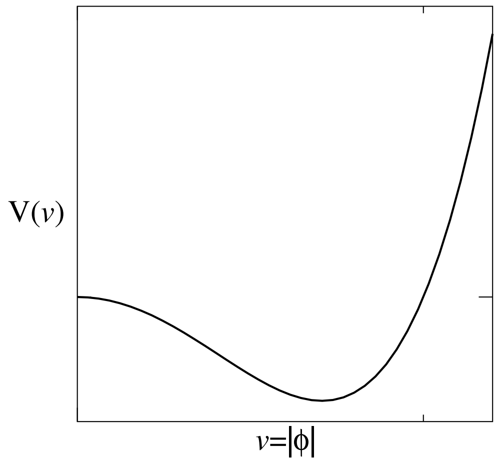


Still, there are many open problems:

- What happens at $T \gtrsim 4T_c$? How does one approach the ideal Stefan-Boltzmann limit?
- Is there a singularity at T_c in the physical case? (Thermodynamic limit.)
- Value of $T_c/\Lambda_{\overline{\mathrm{MS}}}$? (Conversion to physical units and continuum limit.)
- Extrapolation of quark masses to physical values?

EW phase transition

The ground state of the electroweak theory is in a state of a "broken symmetry":



This is not a global symmetry, but a gauge symmetry $\Rightarrow$ there is no gauge-invariant local order parameter, and no strict qualitative difference between the Higgs and confinement phases.

Nevertheless, one may ask: in analogy with condensed matter systems (ferromagnetism, superconductors, etc), is there some sort of a phase transition at high temperatures?

Could be! $T_c \sim$ EW scale ~ 100 GeV.

Kirzhnits, Linde 1972
Weinberg 1974
Dolan, Jackiw 1974

Now coupling is weak, and situation is better than in QCD.

However, a reliable evaluation of the path integral is still difficult because of the multiple length scales related to different types of collective excitations:

$$2\pi T \gg 2gT \gg g^2 T, \quad g \sim \frac{2}{3}.$$

This makes computer simulations of the complete system demanding, since the lattice spacing a and the extent of the lattice Na would have to satisfy

$$a \ll \frac{1}{2\pi T} \ll \frac{1}{2gT} \ll \frac{1}{g^2 T} \ll Na.$$

In addition, chiral fermions are very difficult on the lattice.

The solution:

Combine the best parts of perturbation theory and simulations: integrate out massive modes $(2\pi T, 2gT)$ perturbatively, study light modes (g^2T) non-perturbatively on the lattice — which means using an **effective field theory**.

$\Rightarrow$ **Dimensional reduction into 3d**

& integrate out A_0

Ginsparg; Appelquist, Pisarski; Nadkarni; Reisz; Kajantie, Rummukainen, Shaposhnikov; Jakovác, Patkós; Braaten, Nieto;. . .

The idea of dimensional reduction: write the compact time direction in terms of discrete Fourier (Matsubara) modes.

Fermions: $\omega_n = \pi T(2n+1)$, $n \in \mathbb{Z}$.
Bosons: $\omega_n = 2\pi T n$, $n \in \mathbb{Z}$.

Looking at the structure of the propagators,

$$\frac{1}{\omega_n^2 + |\mathbf{p}|^2},$$

we note that modes with $\omega_n \neq 0$ are effectively massive particles living in 3 Euclidian dimensions. They can be integrated out.

Write then down the most general gauge-invariant effective Lagrangian for the bosonic zero modes, $\omega_n = 0$. For instance, for pure SU(N_c), or QCD,

$$\mathcal{L}_{\text{3d}} = \frac{1}{2}\operatorname{Tr} F_{ij}^2 + \operatorname{Tr}[D_i, A_0]^2 + m_D^2 \operatorname{Tr} A_0^2 + \lambda_A (\operatorname{Tr} A_0^2)^2.$$

Here $m_D^2 \sim g^2 T^2$, $\lambda_A \sim g^4 T$ are generated radiatively, and A_0 can still be integrated out!

To summarise, the modes are:

- typical heavy beyond-the-SM degrees of freedom, as well as all non-zero Matsubara modes, e.g. all fermions: $m \gtrsim \pi T$ (no IR-problems; expansion in $\frac{g^2}{(4\pi)^2}$)

- radiatively generated scale $m \sim gT$ for A_0 (no IR-problems; expansion in $\frac{g}{4\pi}$)

- the non-perturbative scale $m \sim g^2 T$, for purely bosonic Matsubara zero-modes only (no perturbative expansion)

The general form of the effective theory is then

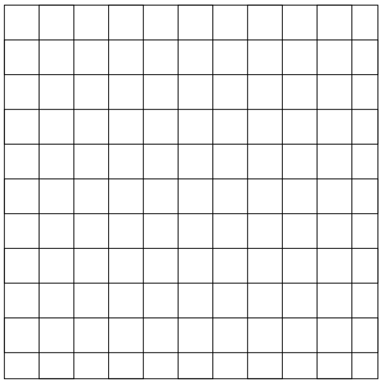
$$\begin{aligned}
\mathcal{L}_{\rm 3d} &= \frac{1}{2}\mathrm{Tr}\, F_{ij}^2 + \frac{1}{4}B_{ij}^2 + (D_i\phi)^\dagger D_i\phi + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2, \\
Z &= \int \mathcal{D}\Phi \exp[-\beta \int d^3x \mathcal{L}_{\rm 3d}^{(\beta)}(\Phi)].
\end{aligned}$$

The couplings, of course, do get modified by the other modes.

$$\begin{aligned}
m_3^2 &= -\frac{1}{2}m_H^2 + cg^2T^2 + ..., \\
\lambda_3 &= \frac{g^2}{8}\frac{m_H^2}{m_W^2} + \ldots .
\end{aligned}$$

Study the 3d theory with lattice simulations!

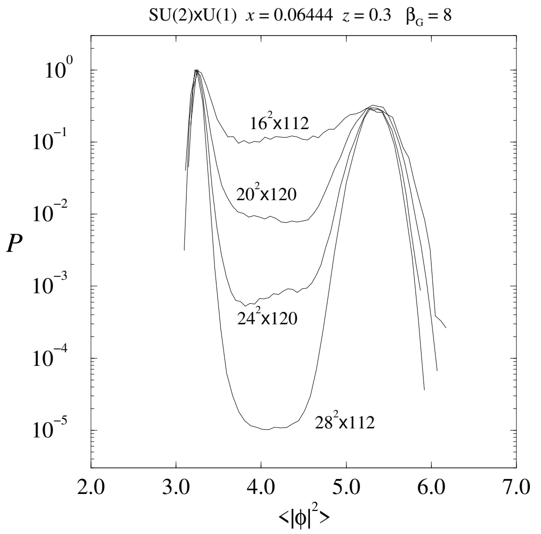
Discretize the space and introduce a finite volume. In the end,

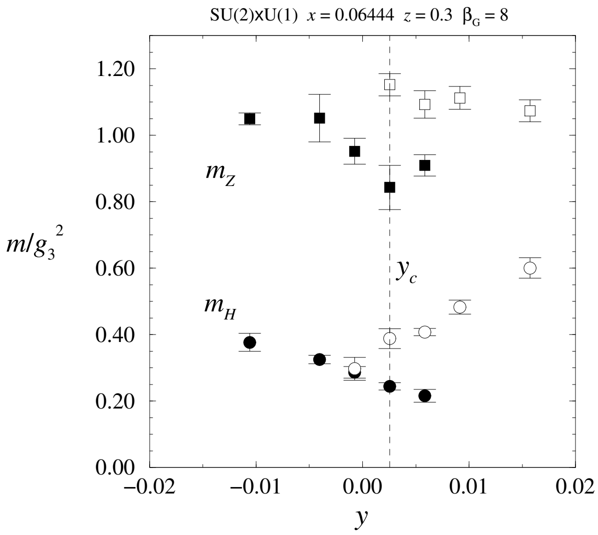


- extrapolate $V \to \infty$,
- extrapolate $a \to 0$.

Due to the fact that the effective theory is super-renormalizable, the continuum extrapolation can be done very systematically.

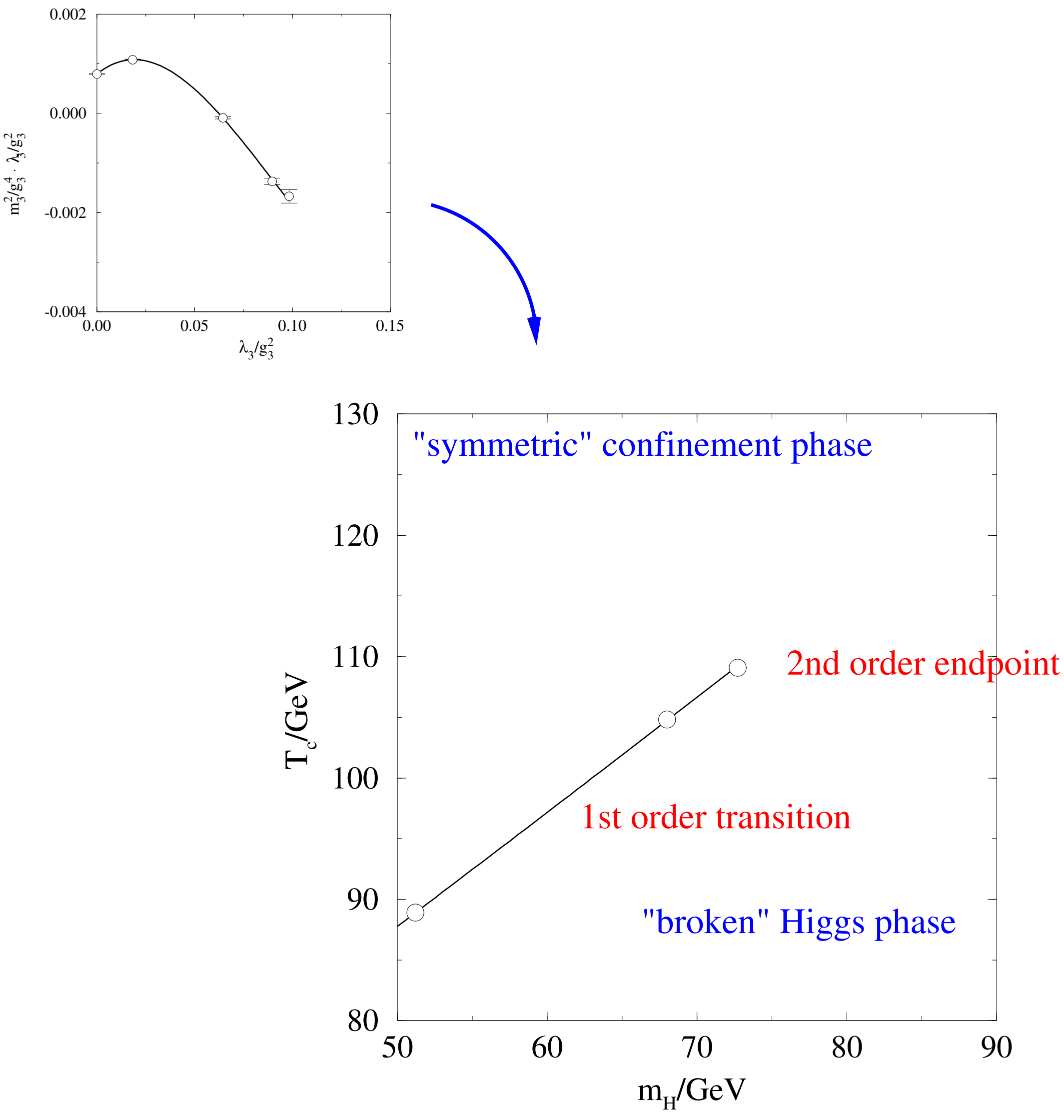
A typical signal for a 1st order transition:





The result for the phase diagram: (after 10^{16} floating point operations)

Kajantie et al; Karsch et al; Gürtler et al; Csikor et al



Like in water-vapour!
The experimental lower limit: $m_H \gtrsim 113$ GeV.

$\Rightarrow$ although all the ingredients for baryogenesis are there already in the Standard Model, in practice there is no phase transition, perfect equilibrium, and hence one needs something more!

Summary and conclusions

Cosmological observations of microwave background and light element abundances agree perfectly with known atomic and nuclear physics, and indicate a thermal Universe at $T \sim 0.1$ eV...1.0 MeV, with three light neutrino species and a global baryon asymmetry.

A priori there is no reason to doubt that the Universe would be thermal even up to $T \sim 10^{15}$ GeV. Then particle physics theories may undergo phase transitions, or leave decoupled weakly interacting relics, and cosmological observations give strong constraints on BSM theories up to these energy scales.

(The large scale structure of the Universe and CMB power spectra give even constraints on physics all the way to the Planck scale $\sim 10^{19}$ GeV, via inflationary dynamics, the cosmological constant, etc.)

Further reading:

S. Weinberg, *Gravitation and Cosmology*; *The First Three Minutes*

E.W. Kolb, M.S. Turner, *The Early Universe*

A. Linde, *Particle Physics and Inflationary Cosmology*

J.I. Kapusta, *Finite-temperature Field Theory*

F. Wilczek, *QCD in Extreme Conditions*, hep-ph/0003183

M. Shaposhnikov, *Cosmology and Astrophysics*, Proceedings of the European School of High Energy Physics, Caramulo, Portugal, 2000 (to appear).

These notes are available at http://cern.ch/laine/gausdal.ps.gz