Vortex phases in condensed matter and cosmology

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1. The phase diagram of the electroweak theory without an external magnetic field

2. Could there be a primordial magnetic field around at the time of the EW transition?

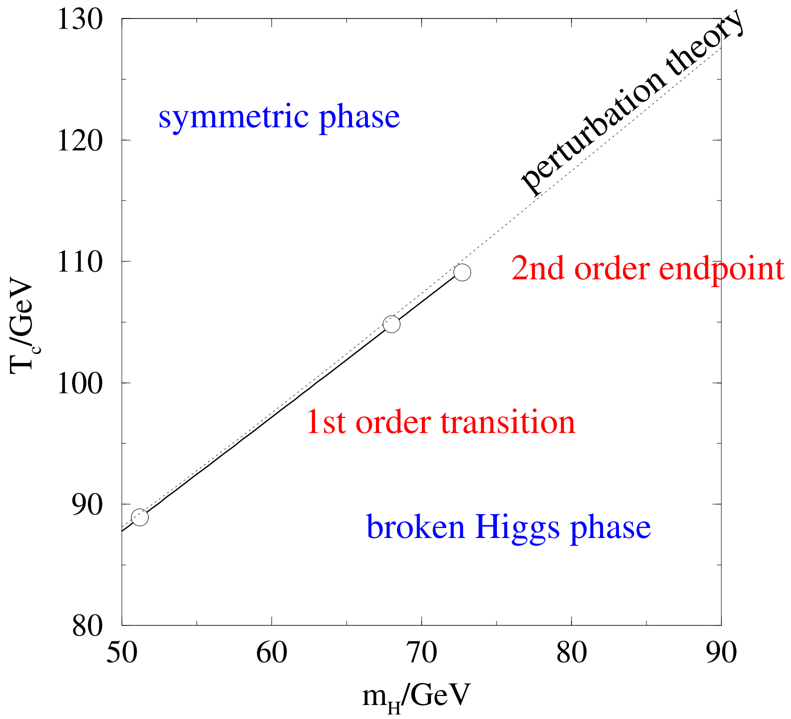
3. Can condensed matter experiments teach us something about the effects of magnetic fields?

4. Lattice results and conclusions

K. Kajantie, M.L., J. Peisa, K. Rummukainen, M. Shaposhnikov, Nucl. Phys. B 544 (1999) 357 [hep-lat/9809004]

K. Kajantie, M.L., T. Neuhaus, A. Rajantie and K. Rummukainen, Nucl. Phys. B, in press [hep-lat/9906028]

The Standard Model phase diagram for $T \neq 0, H = 0$



Kajantie et al 1996;
Karsch et al 1996;
Aoki et al 1996;
Gürtler et al 1997;
Csikor et al 1998

Non-perturbative effects are dominant at realistic Higgs masses.

No phase transition at physical $m_H \Rightarrow$ no cosmological remnant from the EW epoch!

(In the MSSM, larger Higgs masses — up to 110 GeV — can lead to a strong 1st order transition, if there is a light stop.)

The cosmological motivation for magnetic fields:

- The galactic magnetic fields observed today may imply the existence of primordial seed fields.
- In spite of magnetohydrodynamic effects, the seed fields should have a **length scale** larger than the horizon radius around the electroweak era. Son; Enqvist and Olesen
- Such large scale magnetic fields can be generated in the Early Universe? (Inflation, string cosmology,. . .) Turner and Widrow; Ratra; Dolgov and Silk; Gasperini et al;...
- The fields are essentially homogeneous since small length scales diffuse away:

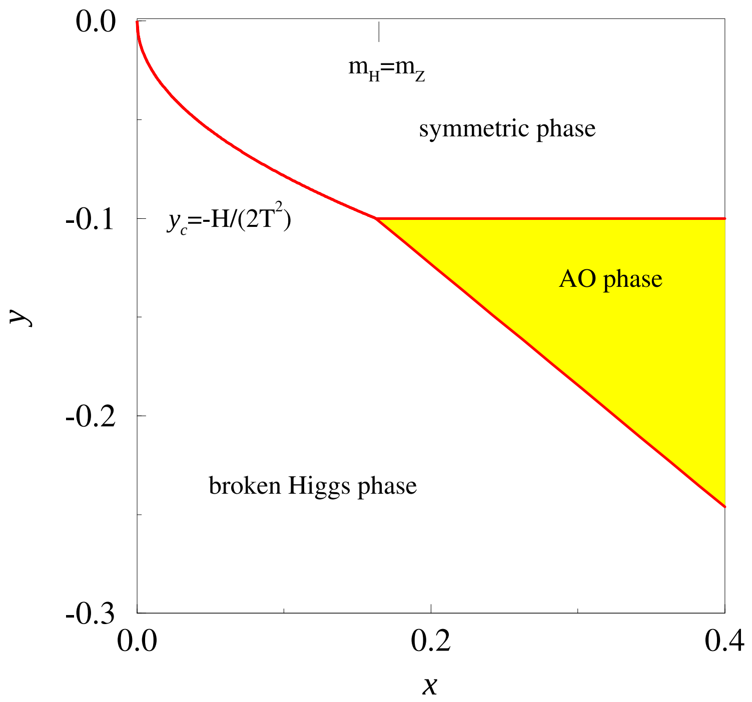
$$l \gtrsim \sqrt{\frac{t}{\sigma}} \sim \frac{1}{T}\sqrt{\frac{M_{\rm Pl}}{T}} \overset{100\mathrm{GeV}}{\sim} 10^7 \frac{1}{T}$$

- **Magnitude**: equipartition tells that only a small fraction of the total free energy can be in magnetic fields $\Rightarrow$

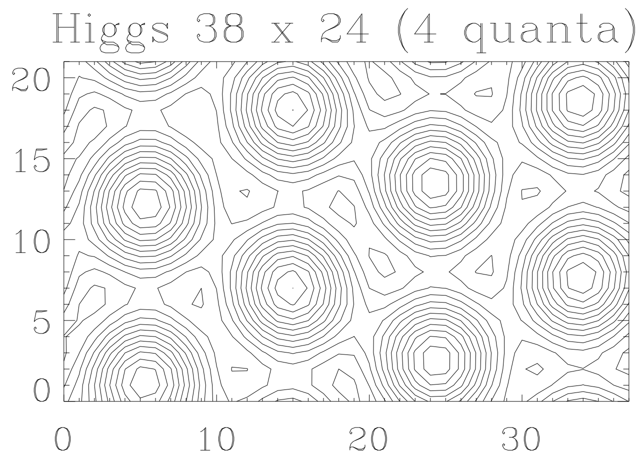
$$\frac{H}{T^2} \lesssim 2.$$

Tree-level phase diagram in a magnetic field:

$$x \sim \frac{1}{8}\frac{m_H^2}{m_W^2}, \quad y \sim 4.5\frac{T-T_0}{T_0}.$$



AO-phase:
Ambjørn, Olesen 1989



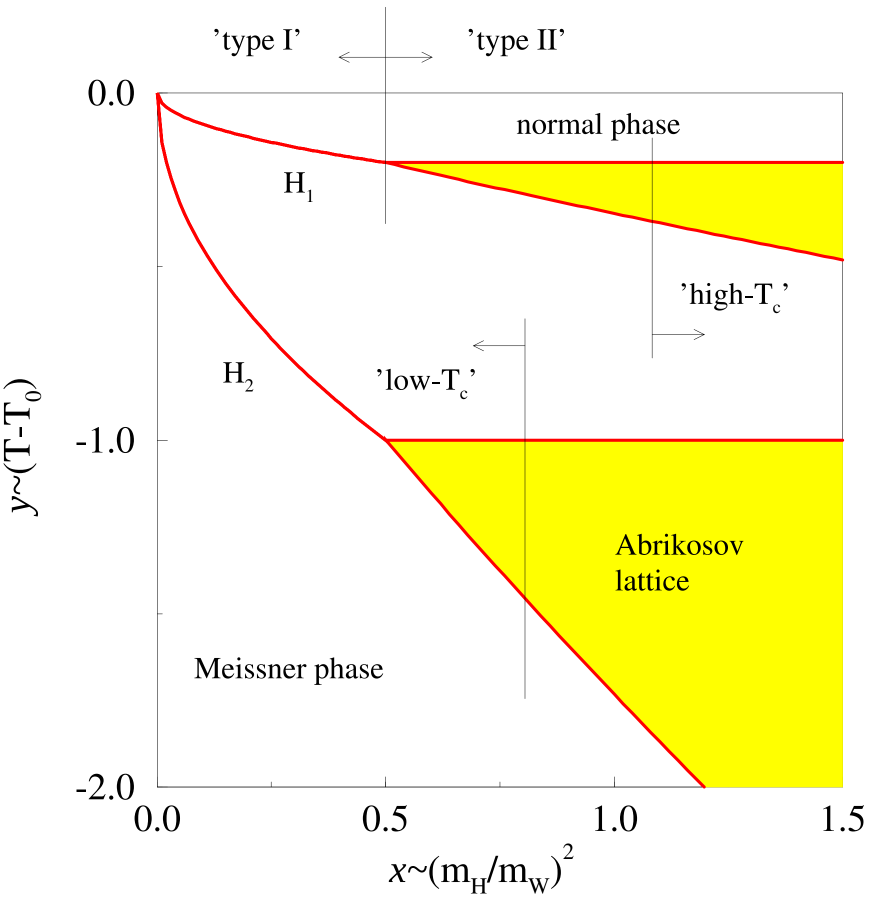
What happens to the AO-phase when fluctuations are included?

We can learn something from condensed matter experiments: the phase diagram of superconductors

The Ginzburg-Landau theory, U(1)+Higgs:

$$\mathcal{L}_{3\mathrm{d}} = \frac{1}{4}F_{ij}F_{ij} + (D_i\phi)^* D_i\phi + m_3^2\phi^*\phi + \lambda_3(\phi^*\phi)^2.$$

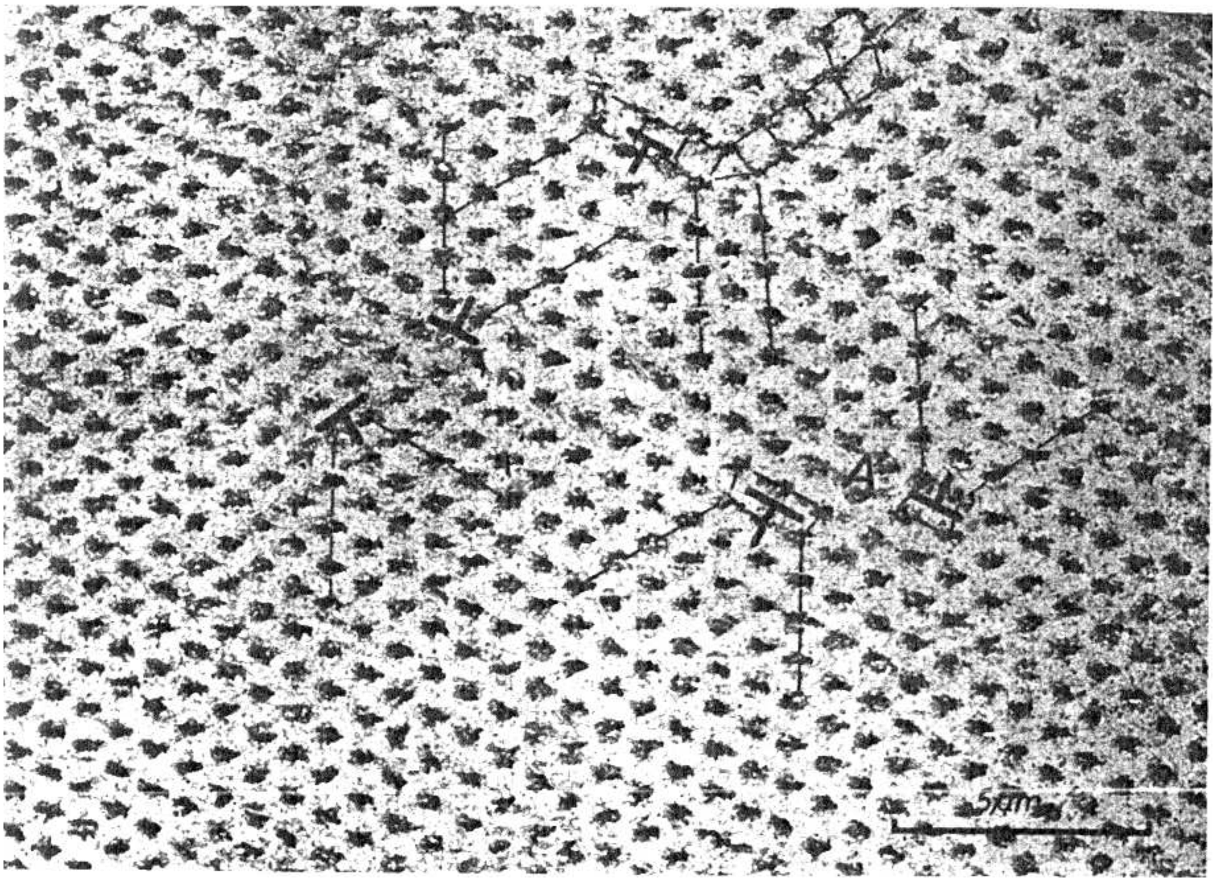
Phase diagram at tree-level:



The Abrikosov lattice can be observed!

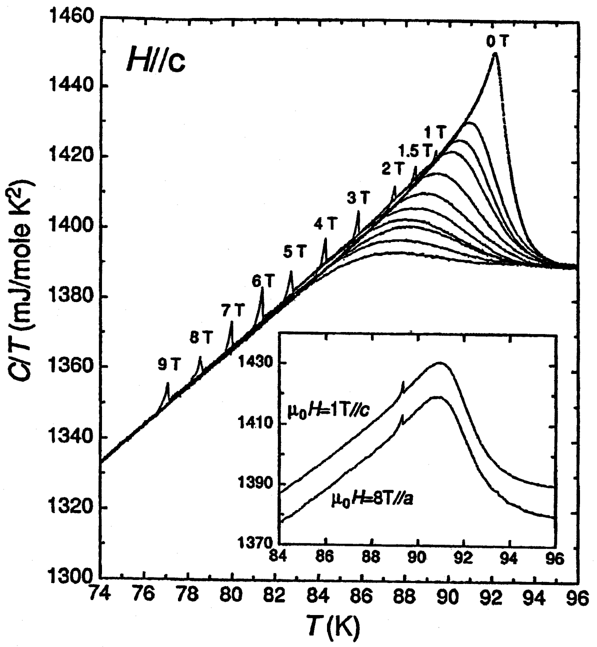
Pb, $T = 1.2$ K, $a \sim 1\ \mu m$.

H. Träuble and U. Essmann,
J. Appl. Phys. 39 (1968) 4052.



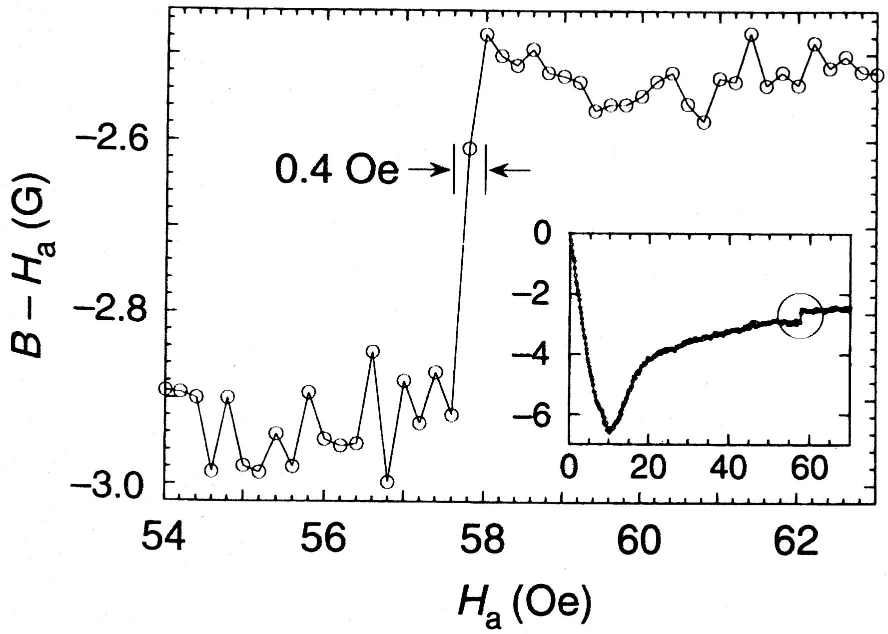
What is even more interesting is that there is a 1st order transition in high-T_c SC's, contrary to tree-level!

Fix H, change T, measure specific heat in 'YBCO':



A. Shilling et al,
PRL 78 (1997) 4833.

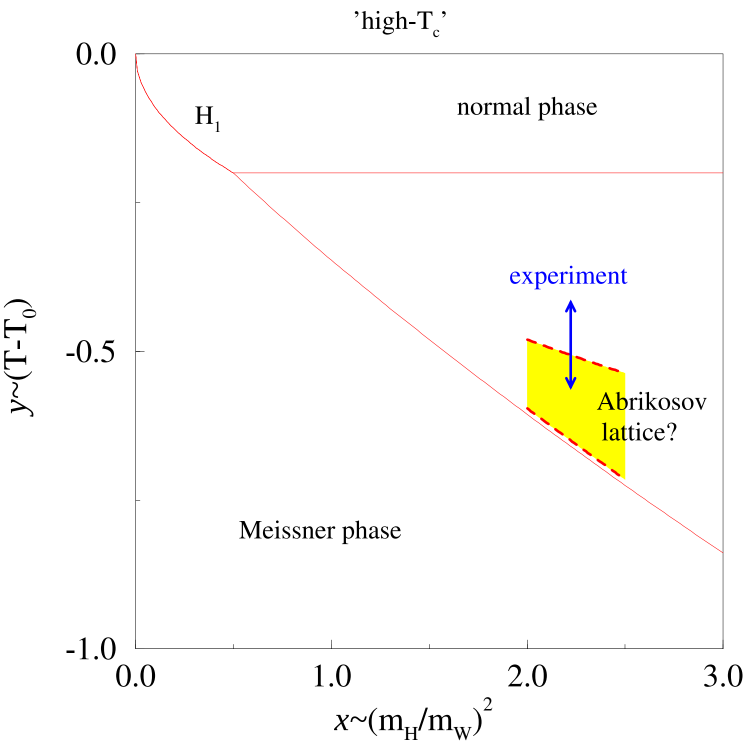
Fix T, change H, measure magnetization in 'BSCCO':



E. Zeldov et al,
Nature 375 (1995) 373.

Interpretation of experiments:

1. How do fluctuations affect the phase diagram? Much of the lattice phase is removed, but a 1st order transition is generated!

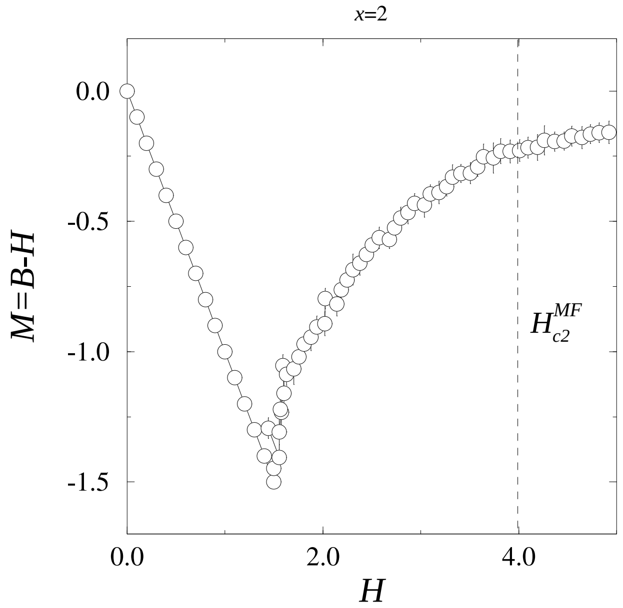


2. Is there really a lattice or a "glass"? This is not known. In any case, there is a transition.

3. What is the universal feature responsible for the transition? In particular, is there a transition in the U(1)+Higgs model, or is a strongly layered structure required? Study the last question with lattice simulations!

Magnetization vs. H in U(1)+Higgs,

result of lattice simulations:



Conclusion: no 1st order signal in U(1)+Higgs!

A layered structure is probably needed. In fact, some people conjecture that the transition is a 'decoupling' transition between different layers, instead of a lattice melting transition.

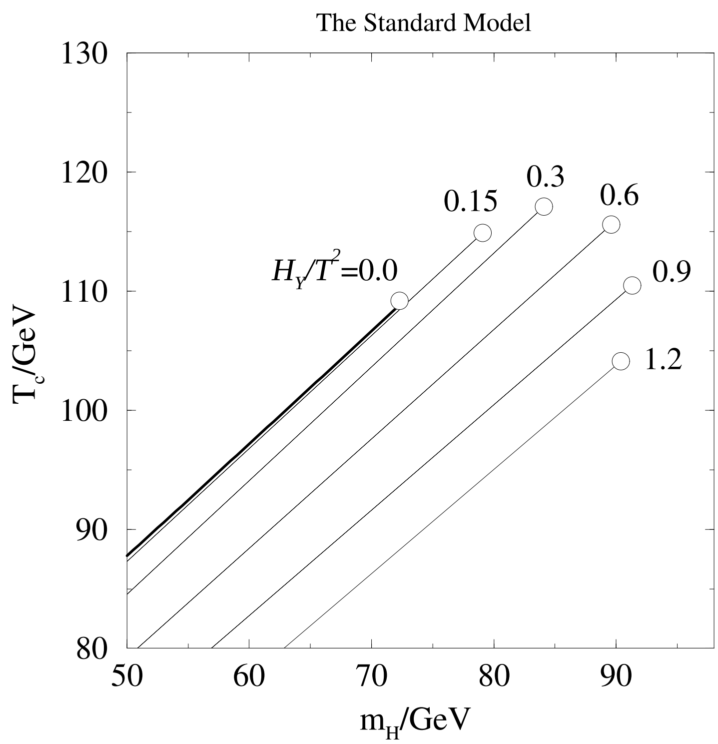
Back to the EW theory

With this background, it is not surprising that in the EW theory, which is even more fluctuation dominated and has no layered structure, no singularities are observed.

The phase diagram can be studied with lattice simulations of

$$\mathcal{L}_{3\mathrm{d}} = \frac{1}{4} G^a_{ij} G^a_{ij} + \frac{1}{4} F_{ij} F_{ij} + (D_i \phi)^\dagger D_i \phi + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2.$$

The final result:



Conclusions:

Even if there is a magnetic field, the SM electroweak transition terminates at $m_H \lesssim 90$ GeV.

The Ambjørn-Olesen phase is not realized at realistic magnetic fields, unlike the Abrikosov 'lattice' in superconductors.

The SM electroweak phase transition leaves no cosmological remnant. Baryogenesis must resort to MSSM, or requires $B-L$ violation, massive ν's,...