# The electroweak phase diagram

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1. **Introduction and motivation**
2. **Methods: effective field theories and lattice simulations**
3. **Results for the phase diagram**
4. **Physics implications**

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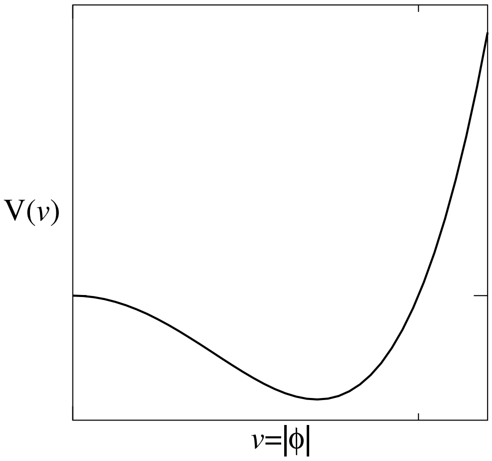
The SU(2)$\times$U(1) electroweak theory of particle physics:

- quarks ("matter particles"; $u, d, c, s, t, b$)
- leptons ("matter particles"; $e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$)
- $W^\pm, Z^0, \gamma$ ("interaction transmitters")
- the Higgs particle

Typical energy scale: $E \sim m_Z c^2 \sim 100$ GeV.

In analogy with condensed matter systems (ferromagnetism, superconductivity, etc), the ground state of the electroweak theory is in a state of a “broken symmetry”:

The Higgs field $\phi$ has an expectation value, $\langle|\phi|\rangle \neq 0$.




The question: in analogy with condensed matter systems, is this symmetry restored at high temperatures? **Yes!**

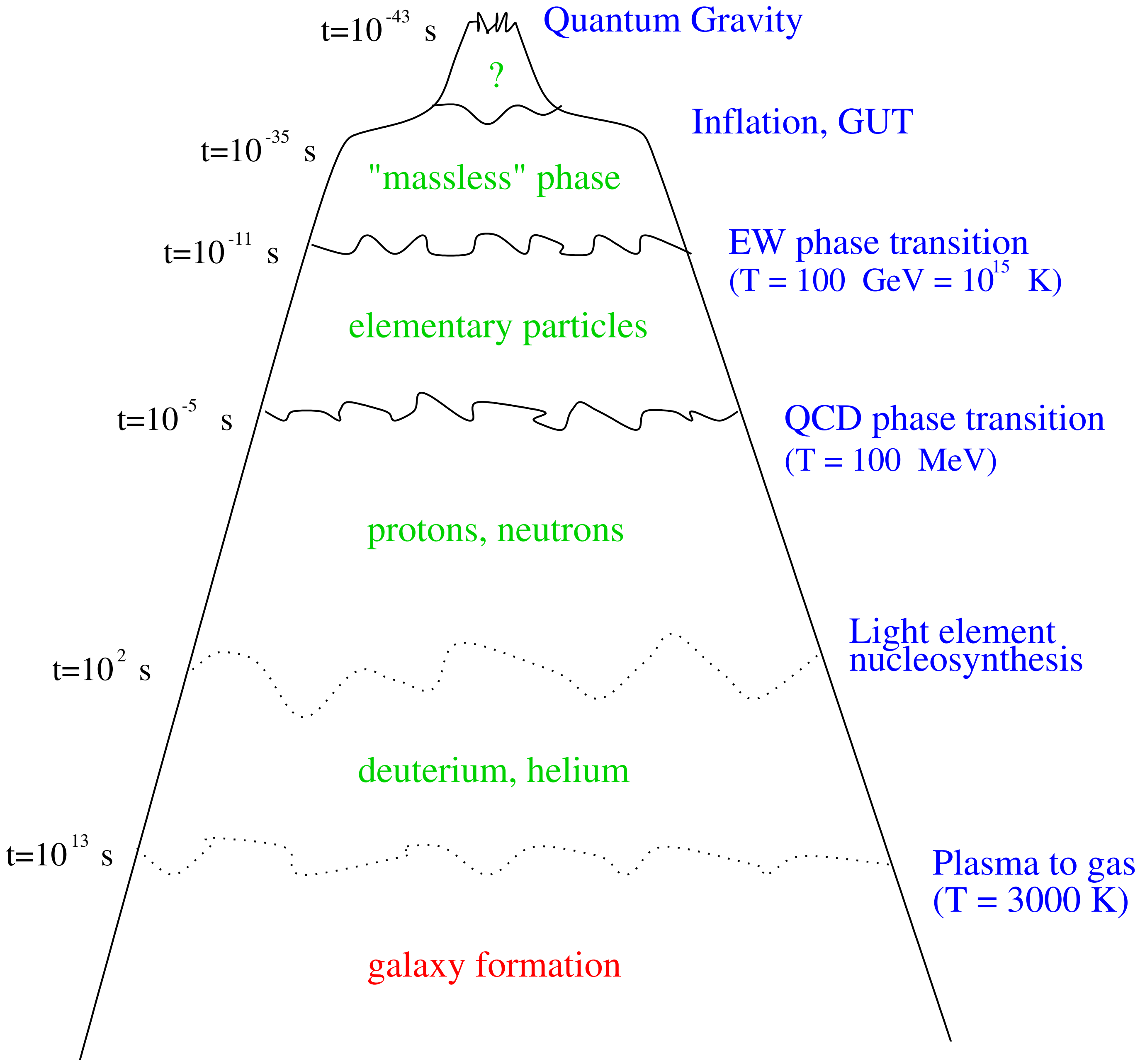
$T_c \sim m_Z c^2 \sim 100$ GeV $\sim 10^{15}$ K.

Kirzhnits, Linde 1972
Weinberg 1974
Dolan, Jackiw 1974

(There is an important subtlety involved: strictly speaking, there is no gauge-invariant order parameter here!)

Such temperatures can exist in the Early Universe!

$$t[s] \approx \frac{1}{(T[10^{10}\ \mathsf{K}])^2}.$$



Suppose that there are phase transitions at the high temperatures appearing in the Early Universe.

# So What?

Do these transitions have observable consequences?

The motivation for studying the electroweak phase transition:
**Generation of Matter–Antimatter Asymmetry**.

Kuzmin, Rubakov, Shaposhnikov 1985

What is matter?

Matter = protons, neutrons, electrons, ... = baryons, leptons.

Quarks: $B = 1/3$.

Leptons: $L = 1$.

In all known reactions, $B, L$ are conserved (e.g. the proton is stable). In perturbation theory, $B, L$ are conserved.

The generation of matter
$\equiv$ The generation of matter–antimatter asymmetry
$\equiv$ The generation of baryon asymmetry
(Neutrinos and thus $L$-asymmetry are difficult to observe!)

The existence of an asymmetry is an experimental fact:

$\star$ there is no antimatter on Earth or in nearby space.

$\star$ no antinuclei are observed in cosmic radiation.

$\star$ there is no observation of significant 150 MeV gamma-radiation, originating from $p\bar{p}$ annihilation.

$\star$ nucleosynthesis (the production of $^4$He, D, Li) $\Rightarrow$

$$\frac{n_B}{n_\gamma} = (1...9) \times 10^{-10},$$

where $n_B$ is the net baryon and $n_\gamma$ is the net photon density. [$n_B \sim 0.1/\text{m}^3$; $n_\gamma \sim 400/\text{cm}^3$].

Nevertheless, it is “natural” that in the very beginning, there was no asymmetry ($B = 0$)!

An important part of the cosmological standard model is **inflation**. Inflation involves a period of very rapid expansion, and is needed to explain why the observed Universe is so homogeneous ($10^{-5}$), even though different regions have apparently not interacted with each other.

The consequence: any initial conditions are diluted away!

$\Rightarrow$ Can $B \neq 0$ be generated after inflation?

**Yes!** The last instance where the asymmetry could have been generated, is the electroweak (EW) phase transition.

Kuzmin, Rubakov, Shaposhnikov 1985

At the same time, this is the possibility requiring the least assumptions beyond established physics, and the only scenario testable at LHC and the Tevatron.

1. There is a quantum mechanical "anomaly" in EW theory!

$$\frac{dB}{dt} \sim \exp(-45\frac{v}{T}).$$

Because of the anomaly, baryon number violation rate is fast enough to be in thermal equilibrium above the electroweak phase transition ($v = 0$), and $B = 0$.

If one now passes continuously to low temperatures, essentially nothing is generated. One needs to have some sort of a singularity. A singularity is provided by a 1st order phase transition, $\Delta v/T \gtrsim 1$, after which there is non-equilibrium.

Shaposhnikov 1987

2. In addition, need CP-violation: $B$ is $P+, C-$.

Quite generally, the theoretical questions concerning the macroscopic thermodynamics of particle physics theories are:

Take Hamiltonian, as determined by accelerator experiments.

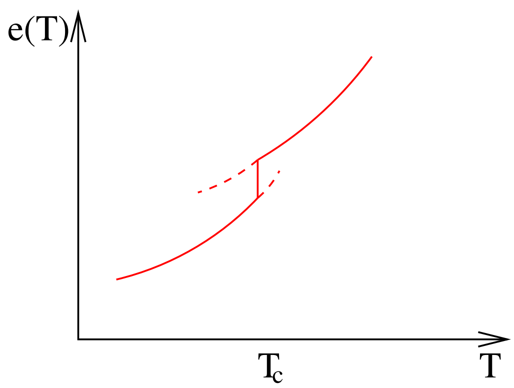
Compute $Z(T,V) = \mathrm{Tr}\exp(-\beta\hat{H}), \quad \beta = T^{-1}$.

We want to know, given $m_H$:

| | |
|---|---|
| Free energy: | $F(T,V) = -T\ln Z(T,V)$. |
| Pressure: | $p(T) = -\frac{\partial F(T,V)}{\partial V} = -\frac{F}{V}$. |
| Entropy density: | $s(T) = \frac{dp}{dT}$. |
| Energy density: | $e(T) = Ts(T) - p(T)$. |
| Expectation values: | $\langle O\rangle(T) = Z^{-1}\,\mathrm{Tr}\,O\exp(-\beta\hat{H})$. |

Correlation lengths, ...

Phase transitions?



How to determine the thermodynamics?

Conceptually, we know what to do:

$$Z = \mathrm{Tr}\exp(-\beta\hat{H}) = \int_{\text{b.c.}} \mathcal{D}\Phi \exp[-\frac{1}{\hbar}\int_0^{\beta\hbar} d\tau \int d^3x \mathcal{L}_E(\Phi)].$$

However, a reliable evaluation of the path integral is difficult because the system has multiple length scales, related to different types of collective excitations:

$$2\pi T \gg 2gT \gg g^2T, \quad g \sim \text{coupling constant} \sim \frac{2}{3}.$$

A standard method in particle physics is **perturbation theory** in $g^2$, but this does not work for the infrared scales $g^2T$.

Linde 1980;
Gross, Pisarski, Yaffe 1981

On the other hand, **non-perturbative** computer simulations of the complete system are demanding, since the lattice spacing $a$ and the extent of the lattice $Na$ would have to satisfy

$$a \ll \frac{1}{2\pi T} \ll \frac{1}{2gT} \ll \frac{1}{g^2T} \ll Na.$$

In addition, chiral fermions are very difficult on the lattice.

The solution:

Combine the best parts of perturbation theory and simulations: integrate out massive modes ($2\pi T, 2gT$) perturbatively, study light modes ($g^2T$) non-perturbatively on the lattice!

**Perturbative part:**
**Dimensional reduction into 3d**

Ginsparg; Appelquist, Pisarski; Nadkarni; Reisz; Kajantie, Rummukainen, Shaposhnikov; Jakovác, Patkós; Braaten, Nieto;. . .

Identify the light degrees of freedom: only bosons remain, due to Bose-Einstein enhancement!

$$\begin{aligned} T = 0 \quad &\Rightarrow \quad g^2 \\ T > 0 \quad &\Rightarrow \quad g^2 n_b(E) = \frac{g^2}{e^{E/T} - 1} \overset{E \ll T}{\sim} \frac{g^2 T}{E}, \\ & \qquad\; g^2 n_f(E) = \frac{g^2}{e^{E/T} + 1} \lesssim g^2. \end{aligned}$$

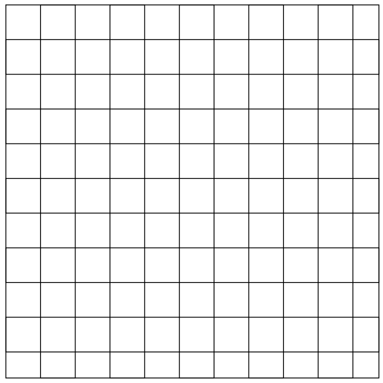
The effective theory left: bosonic "zero modes".

$$\begin{aligned} \mathcal{L}_{\rm 3d} &= \frac{1}{4} G^a_{ij} G^a_{ij} + \frac{1}{4} F_{ij} F_{ij} + (D_i\phi)^\dagger D_i\phi + m_3^2 \phi^\dagger\phi + \lambda_3 (\phi^\dagger\phi)^2, \\ Z &= \int \mathcal{D}\Phi \exp[-\beta \int d^3x \mathcal{L}^{(\beta)}_{\rm 3d}(\Phi)]. \end{aligned}$$

The couplings, of course, do get modified by the other modes.

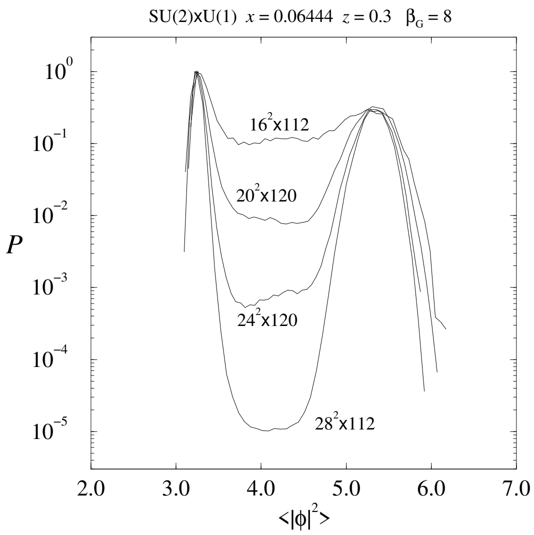
**Non-perturbative part:**
**Study the effective theory with lattice simulations!**

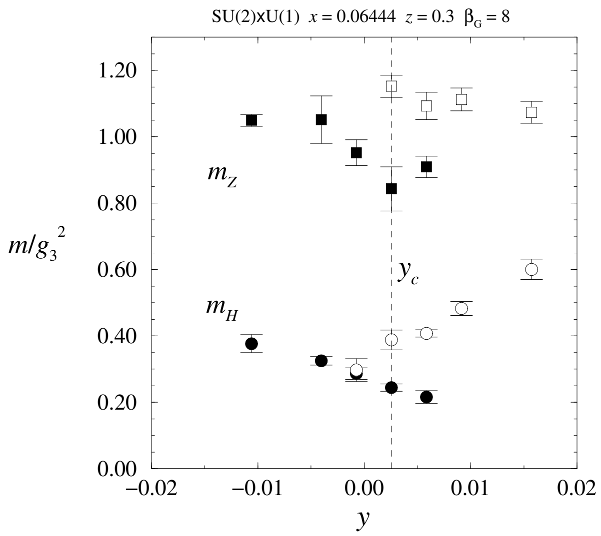
Discretize the space and introduce a finite volume. In the end,



- extrapolate $V \to \infty$,
- extrapolate $a \to 0$.

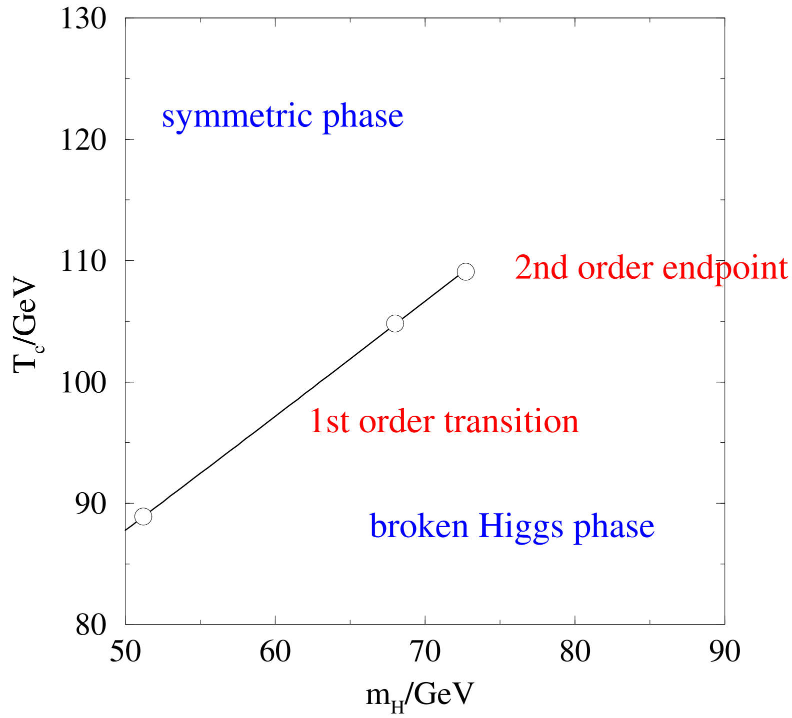
A typical signal for a 1st order transition: $(y \sim (T - T_c)/T_c)$



The result for the phase diagram:
(after $10^{16}$ floating point operations)

Kajantie et al; Karsch et al;
Gürtler et al; Csikor et al




Like in water-vapour!

The experimental lower limit: $m_H \gtrsim 113$ GeV.

$\Rightarrow$ although all the ingredients for baryogenesis are there already in the Standard Model, in practice one needs something more!

What kind of "New Physics" could it be?

1. Change the electroweak theory. For instance, in the Minimal Supersymmetric Standard Model, there are more light bosonic particles, the squarks, which can make the transition stronger.

The unique way: right-handed stop has to be light, $m_{\tilde{t}_R} < m_{\rm top}$, and the lightest Higgs mass is $m_H = 115...120$ GeV.

Carena at al; Delepine et al;
de Carlos and Espinosa;
Cline and Kainulainen;
Losada; Laine; Bödeker et al;
Cline and Moore; ...

2. Change the cosmological history, such that a singularity and "out-of-equilibrium" conditions can be obtained without a phase transition. For instance, inflation very close to the electroweak scale could make baryogenesis work.

Krauss and Trodden;
Garcia-Bellido et al;
Rajantie et al; ...

3. Change the whole mechanism. For instance, certain types of neutrino properties can lead to the generation of a lepton asymmetry, which can be converted to a baryon asymmetry.

Fukugita and Yanagida;
Buchmüller and Plümacher; ...

Etc.

## Conclusions

- $\star$ Particle Physics $\longrightarrow$ Cosmology
  Once a Higgs particle is found, we can systematically do statistical physics in an expanding cosmological background with the corresponding theory, and thus understand the macroscopic properties of the Early Universe.

- $\star$ Cosmology $\longrightarrow$ Particle Physics
  The existence of matter-antimatter asymmetry alone proves that there must exist some unknown, “New” Physics. The asymmetry also places constraints on what that could be.