Primordial magnetic fields and the electroweak phase transition

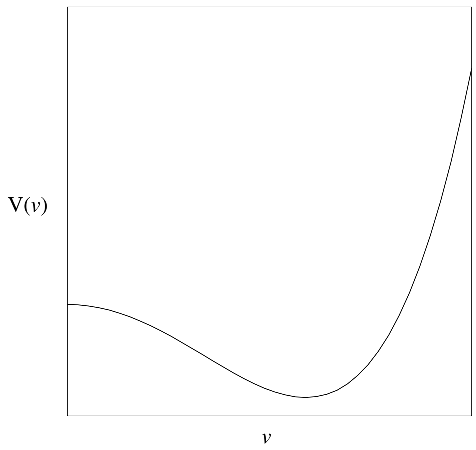
M. Laine (CERN)

1. **Motivation**
2. **Methods**
3. **SM, MSSM without H**
4. **What does an external H do?**
5. **Conclusions**

K. Kajantie, M.L., J. Peisa, K. Rummukainen, M. Shaposhnikov, Nucl. Phys. B 544 (1999) 357 [hep-lat/9809004]

K. Kajantie, M.L., T. Neuhaus, A. Rajantie and K. Rummukainen, Nucl. Phys. B 559 (1999) 395 [hep-lat/9906028]

The ground state of the electroweak theory is in a state of a "broken symmetry":



The question: in analogy with condensed matter systems (ferromagnetism, superconductors, etc), is this symmetry restored at high temperatures? **Yes!**

$T_c \sim m_Z \sim 100$ GeV.

Kirzhnits, Linde 1972
Weinberg 1974
Dolan, Jackiw 1974

There is an important subtlety involved: this is not a global symmetry, but a gauge symmetry $\Rightarrow$ there is no gauge-invariant local order parameter!

Such temperatures can exist in the Early Universe!

So what?

Motivation

The existence of a matter-antimatter (baryon) asymmetry is an experimental fact.

- $\star$ no antinucleons are observed in cosmic radiation.
- $\star$ there is no observation of significant 150 MeV gamma-radiation, originating from $p\bar{p}$ annihilation.
- $\star$ nucleosynthesis (the production of ^{4}He, D, Li) $\Rightarrow$

$$\frac{n_B}{n_\gamma} = (1...9) \times 10^{-10}.$$

Nevertheless, we expect that after inflation $B = 0$.

The electroweak phase transition is the most conservative (lowest energy, latest) scenario for baryon asymmetry generation, and at the same time the only one testable at LEP and the Tevatron.

Kuzmin, Rubakov, Shaposhnikov 1985

The baryon number and the associated symmetry:

$\psi_i \to e^{i\alpha}\psi_i \Rightarrow B+L = \sum_i \int d^3x \bar{\psi}_i \gamma_0 \psi_i$.

With Majorana ν masses, $B-L$ could be violated. In any case, $B+L$ is violated by anomaly. In the following, forget L.

1. Need B-violation: $\langle [B(t)-B(0)]^2 \rangle_T \sim \Gamma V t$.

2. Need CP-violation: B is $P+, C-$.

3. Need non-equilibrium: otherwise,

$$\langle B(t) \rangle_T \propto \mathrm{Tr}\, e^{-H/T} e^{iHt/\hbar} B e^{-iHt/\hbar} = \langle B(0) \rangle.$$

In CMBR, say, the remnant is already there, one simply needs to switch slowly the system out of equilibrium.

Here, if one passes continuously from one regime to the other, essentially nothing is generated. One needs to have some sort of a singularity $\to$ a 1st order phase transition. After it, immediately non-equilibrium.

To determine the thermodynamics, we need to compute

$$Z = \mathrm{Tr}\exp(-\beta H) = \int_{\text{b.c.}} \mathcal{D}\Phi \exp[-\frac{1}{\hbar}\int_0^{\beta\hbar} d\tau \int d^3x \mathcal{L}_E(\Phi)].$$

Method? The problem is non-perturbative!
(even though the theory is weakly coupled and perturbation theory works perfectly at $T = 0$)

This is due to the IR-problem at finite T:
there is a new scale T, and there are new expansion parameters!
This comes from:

$$\begin{aligned} T = 0 \quad &\Rightarrow \quad g^2 \\ T > 0 \quad &\Rightarrow \quad g^2 n_b(E) = \frac{g^2}{e^{E/T} - 1} \overset{E \ll T}{\sim} \frac{g^2 T}{E}, \\ & \qquad\; g^2 n_f(E) = \frac{g^2}{e^{E/T} + 1} \lesssim g^2. \end{aligned}$$

In perturbation theory the transition is between $v = 0$, $v \sim T$, and $m_W \sim gv \Rightarrow$ the expansion breaks down!

At the same time, it is a multi-scale problem!

- non-zero Matsubara modes, e.g. all fermions: $m \sim \pi T$
  (no IR-problems; expansion in $\frac{g^2}{(4\pi)^2}$)

- radiatively generated scale $m \sim gT$ for A_0 (Debye screening)
  (no IR-problems; expansion in $\frac{g}{4\pi}$)

- the non-perturbative scale $m \sim g^2 T$, for purely bosonic Matsubara zero-modes
  (no perturbative expansion)

For such a system, 4d lattice simulations are very demanding, since the lattice spacing a and the extent of the lattice Na would have to satisfy

$$a \ll \frac{1}{3T} \ll \frac{1}{gT} \ll \frac{1}{g^2 T} \ll Na.$$

In addition, (chiral) fermions are hard on the lattice.

The solution:

Combine the best parts of perturbation theory and simulations: integrate out massive modes $(3T, gT)$ perturbatively, study light modes (g^2T) non-perturbatively on the lattice!

This is similar to but more than **universality**: we want to address reliably also non-universal quantities related to a 1st order transition.

$\Rightarrow$ **Dimensional reduction into 3d (& integrate out A_0, too)**

Ginsparg; Appelquist, Pisarski; Nadkarni; Reisz; Kajantie, Rummukainen, Shaposhnikov; Jakovác, Patkós; Braaten, Nieto;. . .

The effective theory left: bosonic zero modes.

$$\mathcal{L}_{3\text{d}} = \frac{1}{4}G^a_{ij}G^a_{ij} + \frac{1}{4}F_{ij}F_{ij} + (D_i\phi)^\dagger D_i\phi + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2,$$

$$Z = \int \mathcal{D}\Phi \exp[-\beta \int d^3x \mathcal{L}^{(\beta)}_{3\text{d}}(\Phi)].$$

The couplings, of course, do get modified by the other modes.

The formal statement: Kajantie et al

All static Green's functions at $T \sim T_c$ can be obtained with relative error

$$\frac{\delta G}{G} \sim \mathcal{O}(g^3)$$

from a 3d theory, when its parameters are suitably fixed. Fixing can be done perturbatively, with a systematic procedure.

The numerical error involved in the reduction step:

$\lesssim 5\%$ in the Standard Model and $\lesssim 10\%$ in the MSSM.

The numerical statement has been recently verified for SU(2)+Higgs with direct 4d simulations. Csikor et al

Study the properties of the 3d theory:

- There is a transition!

$$m_3^2 \sim -m_H^2 + g^2T^2 \Rightarrow T_c \sim \frac{m_H}{g}.$$

- It is of first order!

$$V(v) \sim \frac{1}{2}m_3^2 v^2 - \frac{T}{2\pi}(\frac{gv}{2})^3 + \frac{1}{4}\lambda v^4.$$

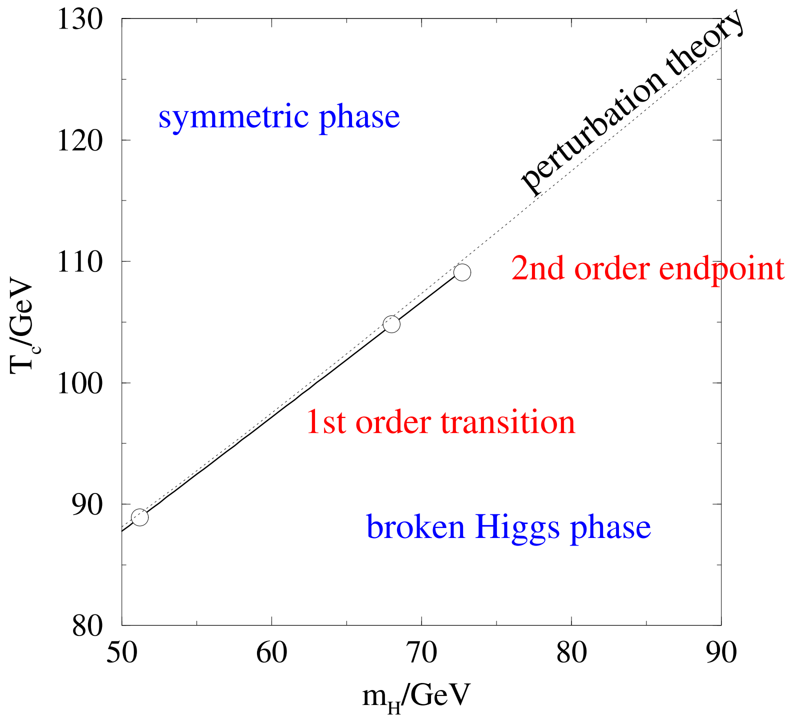
- It gets weaker for increasing λ (or m_H)!

$$\frac{v}{T} \lesssim \frac{3}{16\pi}\frac{g^3}{\lambda} \sim \frac{3g}{2\pi}\frac{m_W^2}{m_H^2}.$$

- Eventually the system is non-perturbative!

$$\text{expansion parameter} = \frac{g^2 T}{\pi m_W} \sim \frac{2gT}{\pi v} \sim \frac{4}{3}\frac{m_H^2}{m_W^2}.$$

The Standard Model phase diagram:

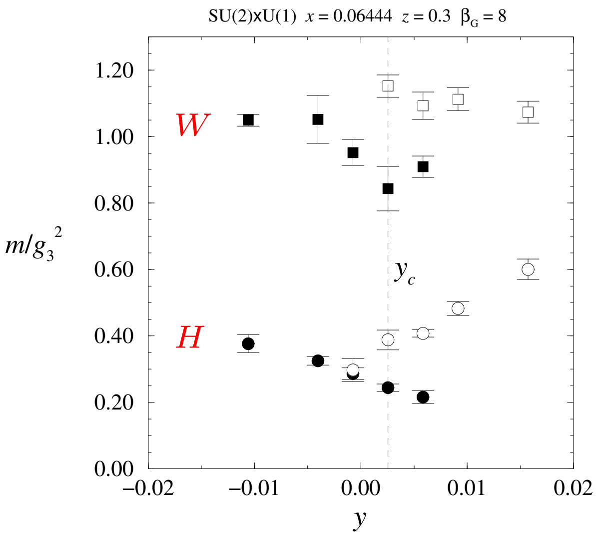


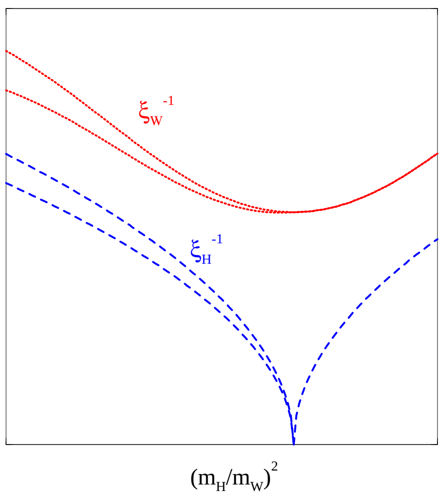
lattice results:
Kajantie et al 1996;
Karsch et al 1996;
Aoki et al 1996;
Gürtler et al 1997;
Rummukainen et al 1998;
Csikor et al 1998

Non-perturbative effects are dominant at realistic Higgs masses.

No phase transition at physical $m_H \Rightarrow$ no cosmological remnant from the EW epoch!

How to understand the form of the phase diagram?
Measure correlation lengths non-perturbatively.

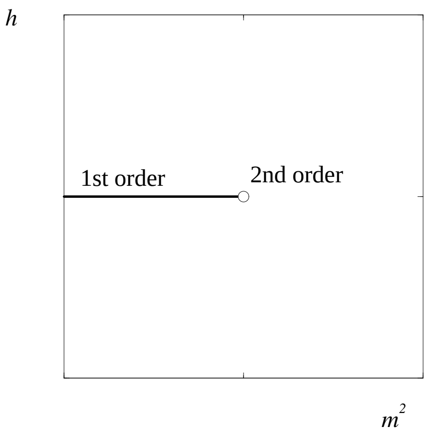




The only divergent degree of freedom is a neutral scalar, the Higgs. In addition, there is a massless U(1) in both phases $\Rightarrow$ close to the endpoint, the effective theory must be ($\phi \in \mathbb{R}$)

$$\mathcal{L} = \frac{1}{4}F_{ij}F_{ij} + \frac{1}{2}(\partial_i\phi)^2 + h\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \gamma_1\phi F_{ij}F_{ij} + \ldots .$$

Here γ_1 is an irrelevant coupling $\Rightarrow$ the universality class is that of the 3d Ising model. The phase diagram:



In the MSSM, in contrast, baryogenesis could work!

1. There can be more CP-violation!
(New phases in the scalar sector.)

Carena et al; Cline et al; Rius and Sanz; ...

2. There can be a stronger transition!
(More light bosonic d.o.f.'s: the squarks.)

Bödeker et al; Carena at al; Cline and Moore; Losada

The regime: right-handed stops are light, $m_{\tilde{t}_R} < m_{\rm top}$
$\Rightarrow$ the Higgs is anything allowed, $m_H \lesssim 110$ GeV.

Lattice simulations show that the transition is even a bit stronger than in 2-loop perturbation theory!

MSSM baryogenesis is a definite scenario, to be tested at LEP and Tevatron during the next few years.

The cosmological motivation for magnetic fields:

- The galactic magnetic fields observed today may imply the existence of primordial seed fields.
- In spite of magnetohydrodynamic effects, the seed fields should have a **length scale** larger than the horizon radius around the electroweak era. Son; Enqvist and Olesen
- Such large scale magnetic fields can be generated in the Early Universe? (Inflation, string cosmology,. . .) Turner and Widrow; Ratra; Dolgov and Silk; Gasperini et al;...
- The fields are essentially homogeneous since the small length scale part of the spectrum diffuses away:

$$\frac{\partial \vec{H}}{\partial t} = \frac{1}{\sigma}\nabla^2 \vec{H} + \nabla \times (\vec{v} \times \vec{H}).$$

$$l \gtrsim \sqrt{\frac{t}{\sigma}} \sim \frac{1}{T}\sqrt{\frac{M_{\rm Pl}}{T}} \overset{100\mathrm{GeV}}{\sim} 10^7 \frac{1}{T} \gg \xi_W, \xi_H$$

- **Magnitude**: equipartition tells that only a small fraction of the total free energy can be in magnetic fields $\Rightarrow$

$$\frac{H}{T^2} \lesssim 2.$$

So we will consider the properties of the EW phase transition in the presence of an external H. To be more precise:

In the symmetric phase, an Abelian U$_Y(1)$ magnetic field.

In the broken phase, this goes dynamically to U$_{\text{EM}}(1)$.

In the simulations, what one does is to impose a hyperflux through the system with boundary conditions:

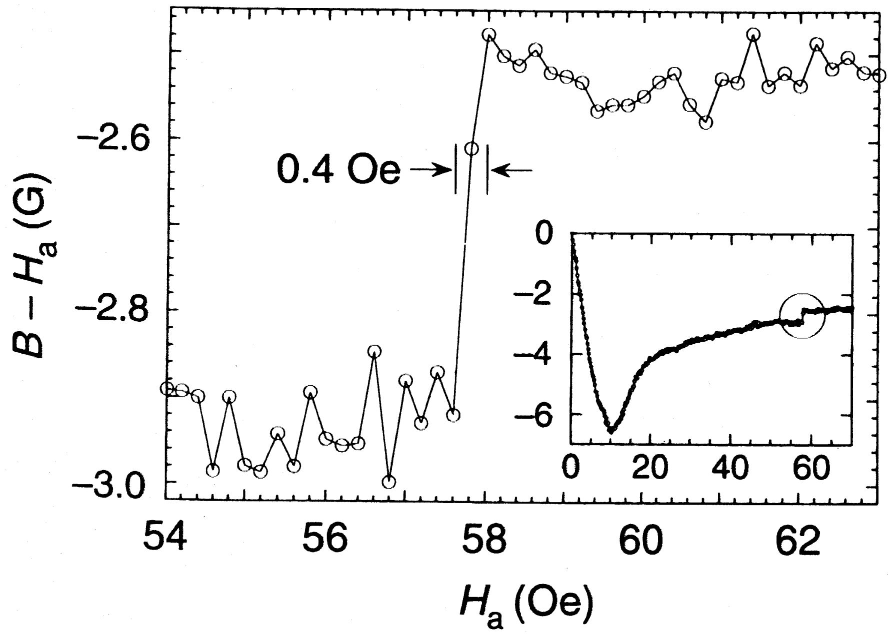
$$\int dx_1 dx_2 F_{12} = \oint ds_i A_i = \Phi_{H_Y}.$$

Qualitative effects of magnetic fields: recall what happens in superconductors.

Small "m_H" (type I): superconductivity is destroyed by H. But a flux cannot go into the SC phase $\Rightarrow$ the normal and SC phase coexist at H_c $\Rightarrow$ a strong 1st order transition!

Large "m_H" (type II): the flux can penetrate the system via an Abrikosov vortex lattice. But even then, there could be a 1st order transition due to vortex interactions:

High-T_c 'BSCCO': fix T, change H, measure magnetization.

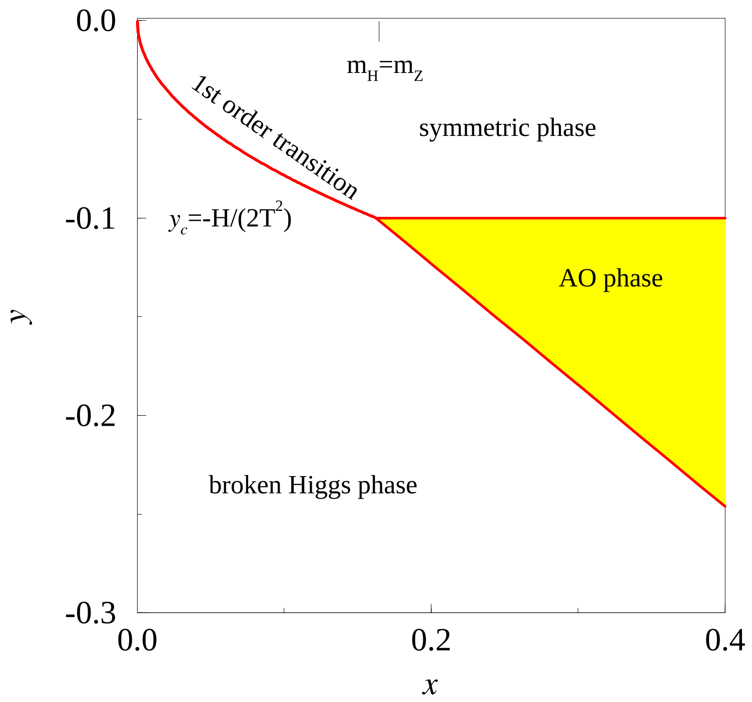


E. Zeldov et al, Nature 375 (1995) 373.

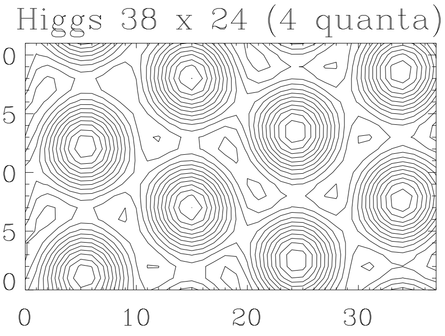
Meissner phase vortex lattice normal (Coulomb) phase

In the electroweak theory: the pattern is surprisingly similar!
At tree-level:

$$x \sim \frac{1}{8}\frac{m_H^2}{m_W^2}, \quad y \sim 4.5\frac{T - T_0}{T_0}.$$



AO-phase:
Ambjørn, Olesen 1989



The way to understand the "instability" is via Landau levels: some excitations become essentially massless, e.g. in the broken phase

$$m^2_{W,\,\text{eff}} = m_W^2 - eH_{\text{EM}} < 0$$

for large enough H, or small enough v.

(For $m_H < m_Z$, the symmetry is restored before the instability sets in).

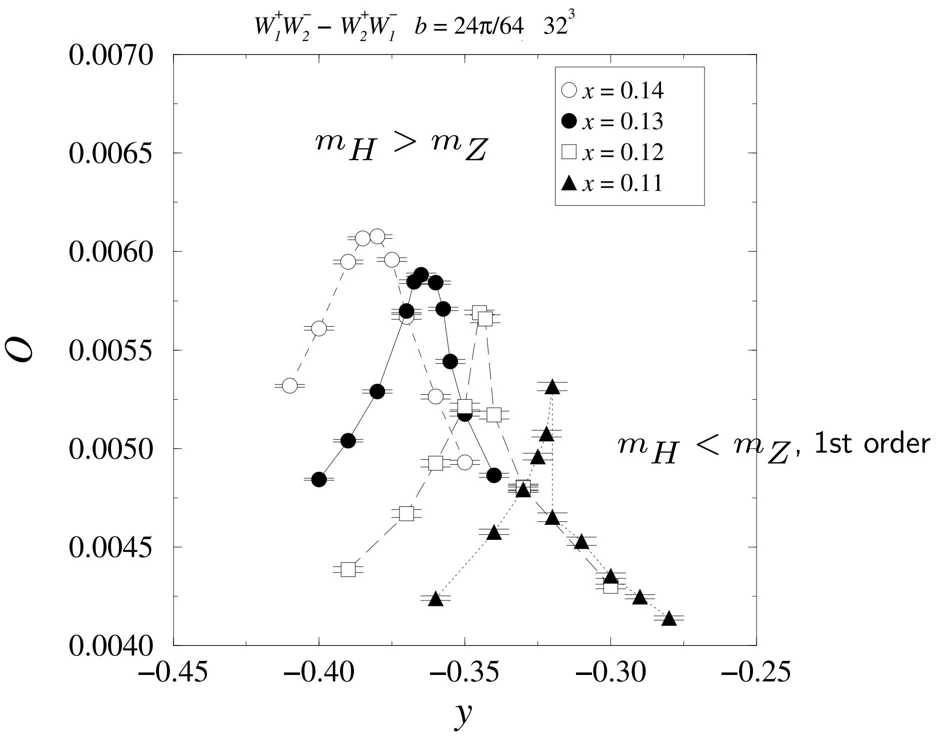
For $m_H < m_Z$: strong 1st order transition, coexisting regions.

[the blob; see `http://www.nordita.dk/~kari/fig13.jpg`]

For $m_H > m_Z$ on the contrary, we do not observe the AO phase, nor any phase transition at all.

For instance, look at W^+W^- condensate which at tree-level signals the AO-phase:

$$W_i^a = \frac{i\,\mathrm{Tr}[\Phi^\dagger D_i \Phi \tau^a]}{\mathrm{Tr}\,\Phi^\dagger\Phi}, \quad O_{W^+W^-} = \frac{\epsilon_{ijk} B_i \langle W_j^+ W_k^- \rangle}{2i|\vec{B}|^2}.$$



A way to understand the absence of the AO-phase: the true light excitations in the final effective theory are not charged!

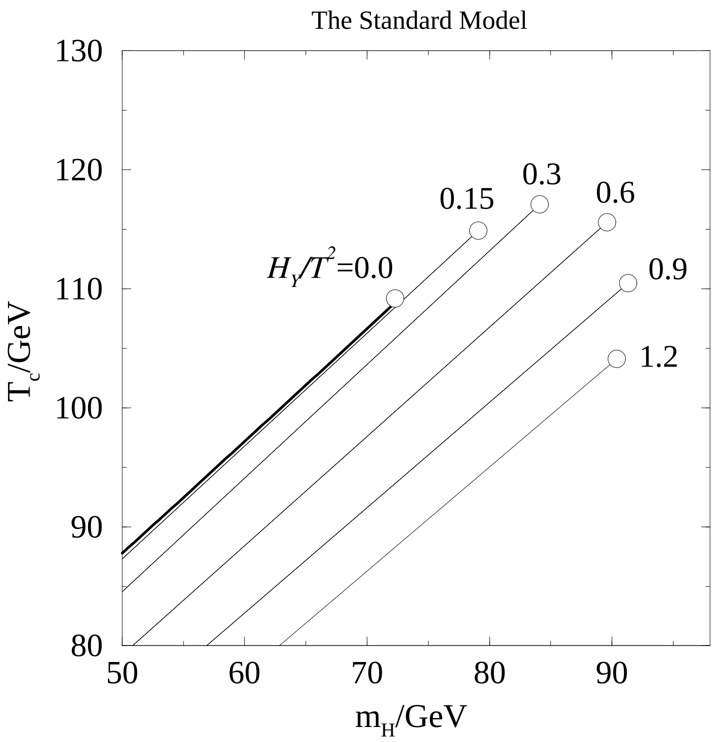
$$\mathcal{L} = \frac{1}{4}F_{ij}F_{ij} + \frac{1}{2}(\partial_i\phi)^2 + h\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \gamma_1\phi F_{ij}F_{ij} + \ldots .$$

No Landau levels! The W, for instance, is always heavy.

However, the effective theory can in principle break down for very large fields, what happens then?

(In fact, we do not seem to find a vortex lattice even in U(1)+Higgs, where there certainly are charged excitations!)

The result so far, for the cosmologically relevant magnitudes:



Conclusions

Electroweak baryogenesis is still an intriguing possibility, even if Majorana ν's would be responsible for part of the result.

Nevertheless, baryogenesis does not work in the SM, even if there were external magnetic fields present.

Could work in the MSSM if $m_H \lesssim 110$ GeV is discovered.

There are many interesting connections between the two cosmological remnants, magnetic fields and baryon asymmetry. Maybe also their generation mechanisms are connected?