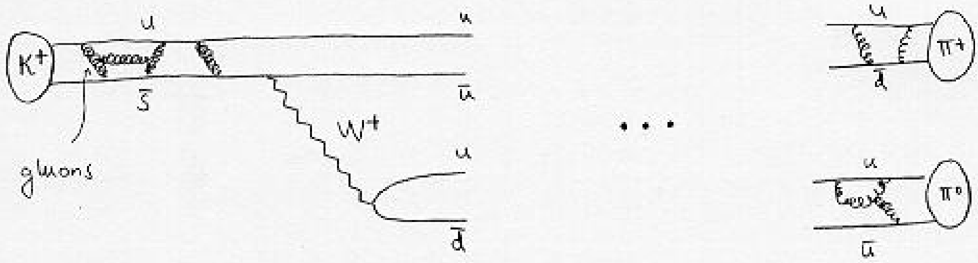
QCD: weak matrix elements

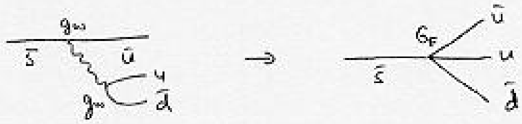
Most of the hadrons are not "stable" particles: they decay.
The decay can be via strong interactions (e.g. $\rho \to \pi\pi$),
weak interactions (e.g. $\pi^\pm \to \mu^\pm \nu_\mu$, $K^\pm \to \mu^\pm \nu_\mu$, $K^\pm \to \pi^\pm \pi^0$, $K^0 \to \pi^+\pi^-, \pi^0\pi^0$), or
"strong + electromagnetic" (e.g. $\pi^0 \to \gamma\gamma$).
[really: "axial anomaly".]

An important problem for lattice QCD is to study weak decays, where both the initial and final state are hadrons. Understanding such decays is a test both of QCD and of the theory of weak interactions (the Weinberg-Salam model).

Consider, in particular, the decays $K \to \pi\pi$.

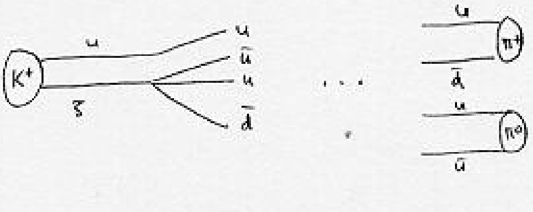


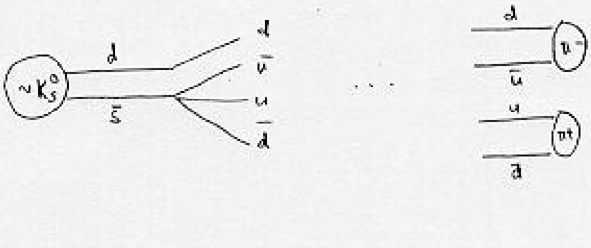
Or, in the local Fermi model:



, G_F = Fermi coupling = $\frac{g_w^2}{4\sqrt{2}\, m_W^2}$

$\Rightarrow$





or $\pi^0\pi^0$.

The mystery :

According to Particle Data Group, the lifetime of $K^\pm$ is $\tau \approx 1.2 \times 10^{-8}$s, that of K_S^0 is $0.89 \cdot 10^{-10}$ s. Moreover for $K^\pm$ only 20% go into $\pi^\pm \pi^0$.

$\Rightarrow$ there is a difference in the decay rates of a factor of several hundred, although the weak processes are the same and the QCD-processes look very similar!

So can the big difference be explained by a more precise study of the QCD-processes involved?

What would one like to do in principle?

- The Fermi-model : $S_W = \int d^4x \, \mathcal{H}_W$,

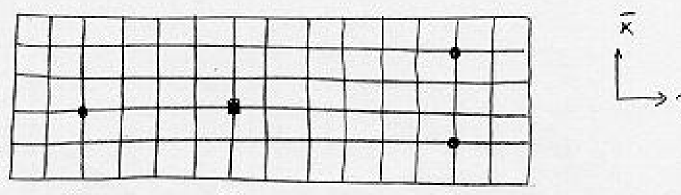
$$\mathcal{H}_W = 2\sqrt{2}\, G_F V_{ud} V_{us}^* \left[(\bar{s}\gamma_\mu P_L u)(\bar{u}\gamma_\mu P_L d) - (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma_\mu P_L d) \right] + \text{h.c.} ,$$

where $V_{ij} \in \mathbb{C}$ are known ~angular parameters $(|V_{ij}| < 1)$ appearing in the so-called CKM-matrix.

- We know how to write operators for K, π in QCD.

↳ Could we just not measure "weak matrix elements" to probe the strength of the decays?

$$\langle K(x) \, \mathcal{H}_W(y) \, \pi(z_1) \, \pi(z_2) \rangle \quad ?$$



?

Challenges for lattice - QCD:

- In Nature, $m_K \approx 495$ MeV, $m_\pi \approx 135$ MeV.
  But on the lattice, one currently has $m_\pi \gtrsim 0.6 \cdot m_g \approx 460$ MeV. (p. 119)

  $\Rightarrow 2m_\pi > m_K \Rightarrow$ kaons cannot decay into two pions!

  Thus, it is essential to approach the "chiral limit" of very light m_u, m_d.

- Nature is Minkowskian, lattice is Euclidean!
  This means that kinematics will not be correct on the lattice, in general. In a way, "energy is not conserved".

  One way to express this:

$$\langle K(\tau_0) | \mathcal{H}_w(\tau_1) | \pi\pi(\tau_2) \rangle$$

$$\sim \langle K \, | e^{-H(\tau_1-\tau_0)} \mathcal{H}_w \, e^{-H(\tau_2-\tau_1)} | \pi\pi \rangle$$

$$= \sum_n e^{-E_K(\tau_1-\tau_0)} \langle K | \mathcal{H}_w | n \rangle \, e^{-E_n(\tau_2-\tau_1)} \langle n | \pi\pi \rangle$$

  $\Rightarrow$ for $\tau_2 - \tau_1 \gg 0$, and infinite spatial volume, Correlator is dominated by the "ground state" with two pions at ~ rest, rather than the proper final state with two pions with opposite momenta $|\bar{p}| = \frac{1}{2}\sqrt{m_K^2 - 4m_\pi^2}$!

- The operator $\mathcal{H}_w$ has a certain chiral structure (since it contains P_L's: only left-handed quarks participate in weak interactions) and a flavour structure (u,d,s,c - quarks appear). Thus it is important to respect the corresponding symmetries $\Rightarrow$ should use the numerically expensive Ginsparg-Wilson fermions.

A principal way around the first two challenges :

Like for the light hadron spectrum, let us again factorise the problem into two parts :

* Write down the analogue of $\mathcal{H}_w$ in the effective theory for the almost - Goldstone modes! It will involve some new effective couplings, often denoted e.g. by g_{27} :

$$\mathcal{H}_w^{(\text{eff})} = 2\sqrt{2}\, G_F V_{ud} V_{us}^* \left\{ g_{27}\, \mathcal{O}_{27} + \ldots \right\} + \text{H.c.},$$

$$\mathcal{O}_{27} = \frac{F^4}{4} \left(\partial_\mu U U^+\right)_{us} \left(\partial_\mu U U^+\right)_{du} + \ldots$$

refers to irreducible representation of the flavour group.

Use then the effective theory to approach the limit of light quark masses and to handle analytically (after continuation to Minkowski space) correctly the kinematics.

* Like for F^2 (p. 123), use lattice QCD to match for the coefficient g_{27} ! This can be done in Euclidean space, and even at finite volume ! (p. 124). Now we can use any three-point correlator, like $\langle K(x)\, \mathcal{H}_w(y)\, \pi(z) \rangle$!

Easy to state [Bernard, Draper, Soni, Politzer, Wise, Phys. Rev. D 32 (1985) 2343], difficult to realise in practice because of the third challenge on p. 128 ...

Summary:

- Apart from "spectroscopy", or study of two-point correlators, another main direction for lattice QCD is the study of weak decay rates, via measurement of three-point correlators.

- One is faced with the same problem of the scale hierarchy as before, but also many others, such as wrong "final-state kinematics" and urgent need to respect chiral and flavour symmetries.

- The general philosophy of matching to effective low-energy theories is again a good way ahead.

- Definite physical results will only be obtained in the future.