QCD: spectroscopy, directly: hadronic correlation functions; problem of scale hierarchies

Previous lecture $\Rightarrow$ a good candidate for an operator coupling strongly to very light (almost Goldstone) mesons, is $P^a(x) \equiv \bar{\psi}_i T^a_{ij} i\gamma_5 \psi_j$, where T^a is a generator of $SU(N_f)$, and $N_f = 3$.

For instance: $T^{\circ}_{ij} = \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \Rightarrow P^a(x) \equiv \pi^0(x) \equiv \frac{1}{2}\{\bar{u} i\gamma_5 u - \bar{d} i\gamma_5 d\}$.

A typical <u>non-Goldstone</u> operator: $\bar{\psi}_i T^a_{ij} \gamma_\mu \psi_j \Rightarrow V^a_0(x) \equiv \rho(x) \equiv \frac{1}{2}\{\bar{u}\gamma_0 u - \bar{d}\gamma_0 d\}$.

<u>Nucleons</u> $(p, n, \cdots)$ are of the type $N(x) = \sum_{\alpha,\beta,\gamma} \sum_{i,j,k} C_{\alpha\beta\gamma,ijk} \sum_{ABC} \epsilon_{ABC} \psi_{iA\alpha}(x) \psi_{jB\beta}(x) \psi_{kC\gamma}(x)$,

where $C_{\alpha\beta\gamma,ijk}$ are numerical coefficients, and ϵ_{ABC} is the antisymmetric Levi-Civita tensor, guaranteeing gauge invariance ($\epsilon_{ABC}\, g_{AA'} g_{BB'} g_{CC'} = \underbrace{\det(g)}_{1}\, \epsilon_{A'B'C'}$).

<u>Correlation functions:</u>

- For mesons: $\langle O(x)\, O(y) \rangle = \langle \bar{\psi}_i(x) \Gamma_{ij} \psi_j(x)\, \bar{\psi}_k(y) \Gamma_{kl} \psi_l(y) \rangle$

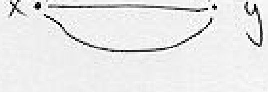
$$= -\, \underbrace{\psi_l(y)\bar{\psi}_i(x)}\Gamma_{ij}\, \underbrace{\psi_j(x)\bar{\psi}_k(y)}\Gamma_{kl}$$

$$+\, \underbrace{\psi_j(x)\bar{\psi}_i(x)}\Gamma_{ij}\, \underbrace{\psi_l(y)\bar{\psi}_k(y)}\Gamma_{kl}$$

Fermion propagators e.g. through spectral representation (p. 98: $\underbrace{\psi(x)\bar{\psi}(y)} = \sum_k \frac{v_k(x) v_k^+(y)}{\lambda_k}$)

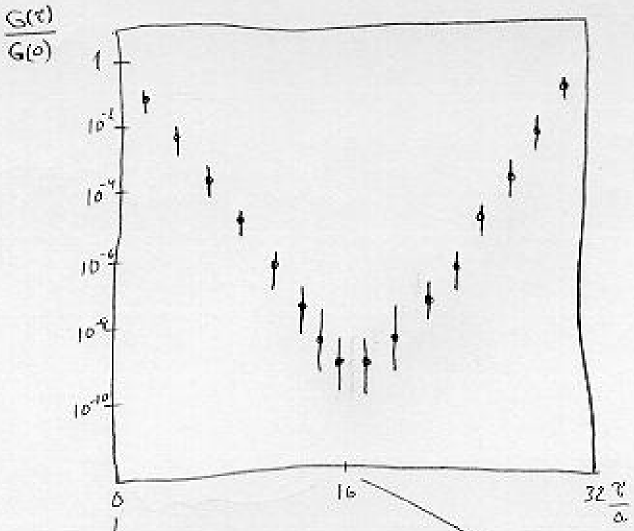
- For nucleons: $\langle O(x)\, O^+(y) \rangle = \langle \Gamma_{ijk} \psi_i(x)\psi_j(x)\psi_k(x)\, \Gamma_{lmn} \bar{\psi}_l(y) \bar{\psi}_m(y) \bar{\psi}_n(y) \rangle$

$=$



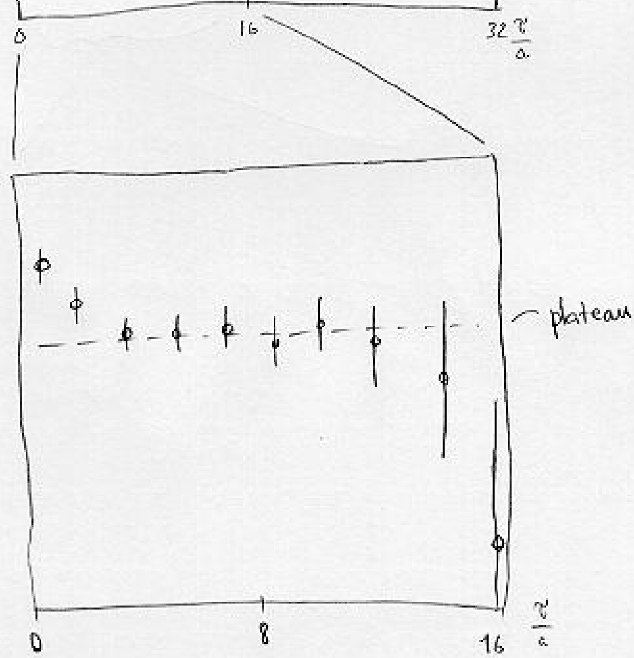
Masses are determined from exponential falloff of correlators, $\sim e^{-m\tau}$.

Raw data:



Local masses:

$$ma = \ln \frac{G(\tau)}{G(\tau+a)}$$



Parameters:

- $\beta_G = \frac{2N_c}{g_0^2}$, determines the lattice spacing.
  E.g. (p. 104): $\beta_G = 6.0 \Leftrightarrow a \approx 0.1$ fm.
  Several β_G's should be studied.

- Volume, $N_x \cdot N_y N_z N_\tau$, e.g. $16^3 \times 32 \ldots 32^3 \times 64$.
  Several volumes should be studied.

- Quark masses, $m_{u,0}$, $m_{d,0}$, $m_{s,0}$. For any given β_G, should be tuned so that experimentally observed values are observed for the hadrons!

It turns out that decreasing the quark masses m_u, m_d to small enough values, is technically extremely demanding!

In physical terms, this is due a <u>scale hierarchy</u>.

$$m_\pi \approx 140\ \text{MeV} \quad \ll \quad m_{\text{glueball}} \approx 1700\ \text{MeV}$$

or: $$m_u, m_d \sim \mathcal{O}(10)\ \text{MeV} \quad \ll \quad \text{QCD-scale} \approx \text{a few hundred MeV}.$$

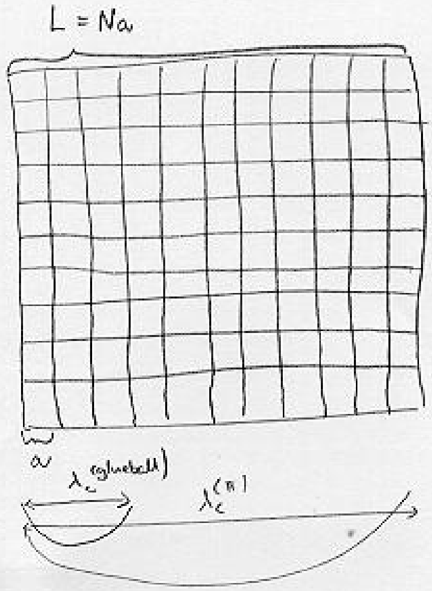
Compton wavelength: $\lambda_c = \frac{2\pi\hbar}{mc} \underset{\hbar = c = 1}{=} \frac{2\pi}{m}$

For lattice to be close to continuum limit, lattice spacing should be <u>smaller</u> than the smallest physical length scale in the system, given by the heaviest particle's Compton wavelength.

For lattice to be large enough to be close to infinite volume limit, it needs to be <u>larger</u> than the largest physical length scale.

$$\text{GeV} \cdot \text{fm} = 5 \quad \Rightarrow \quad \lambda_c^{(\text{glueball})} \approx \frac{2\pi}{1.7\ \text{GeV}} = 0.7\ \text{fm}$$

$$\lambda_c^{(\pi)} \approx \frac{2\pi}{0.14\ \text{GeV}} \approx 9.0\ \text{fm}$$



$$a = 0.1\ \text{fm} \ll \lambda_c^{(\text{glueball})} \approx 0.7\ \text{fm} \ll \lambda_c^{(\pi)} \approx 9.0\ \text{fm} \ll L = Na$$

$$\Rightarrow \underline{N \gg 100!} \quad \Rightarrow \quad \text{not accessible at present!}$$

(typical "large" lattice $\sim 32^3 \times 64$)

Same problem more technically (one flavour):

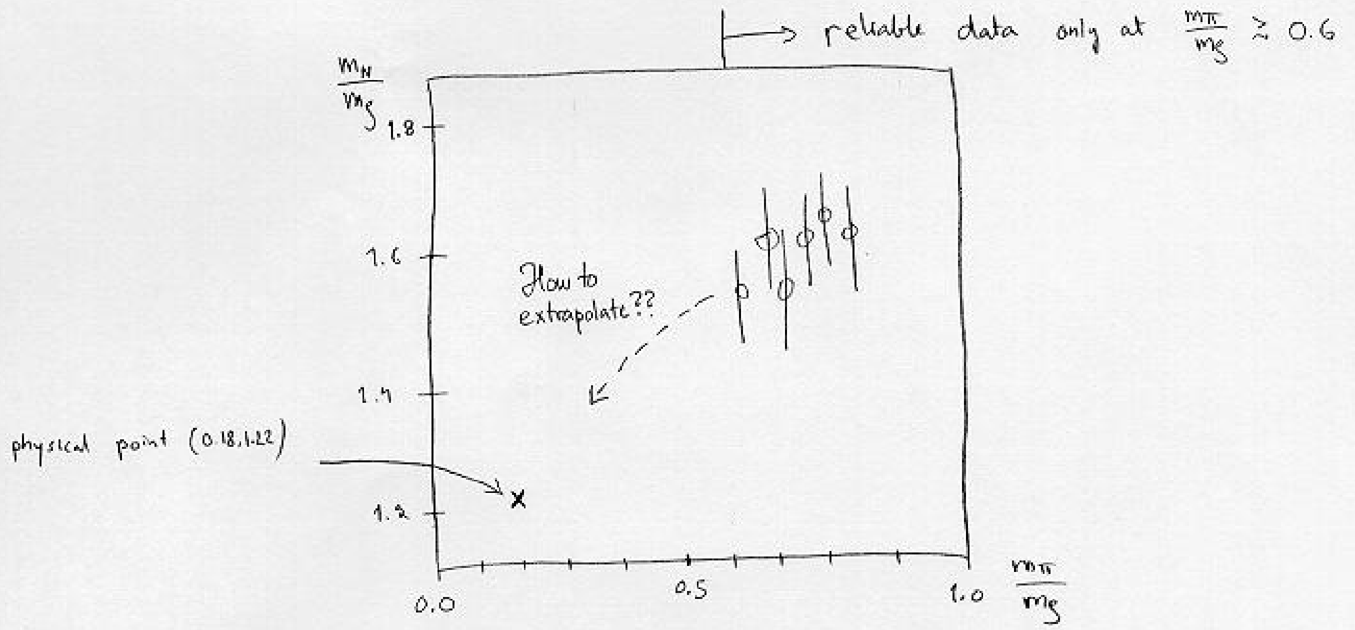
$$\underbrace{\psi(x)\bar{\psi}(y)} = \sum_k \frac{v_k(x)v_k^\dagger(y)}{\lambda_k} \quad , \quad (\not{D}+m)\,v_k(x) = \lambda_k v_k(x)$$

$\Rightarrow\ |\lambda| \gtrsim |m|$

$\Rightarrow$ if $m \to 0$:

- $\frac{1}{\lambda}$ is large, and $\lambda, v(x)$ have to be solved for very precisely.
- $\frac{1}{\lambda}$ can fluctuate much between various gauge field configurations, leading to a bad signal ($\equiv$ large statistical error).

Status : physical masses $m_\pi \simeq 140$ MeV, $m_\rho \simeq 770$ MeV, $m_N \simeq 940$ MeV . Plot $\frac{m_N}{m_\rho}$ vs. $\frac{m_\pi}{m_\rho}$.



Summary:

- In principle we can write down local operators in QCD, which correspond to the almost-Goldstone bosons of chiral symmetry breaking ($\pi^\pm, \pi^0$), to non-Goldstone mesons ($\rho, \ldots$), or to nucleons ($p, n, \ldots$).

- The masses of such objects can be determined from first principles by measuring the exponential falloff of the corresponding two-point correlation functions.

- Due to various difficulties, related to treating systems with a scale hierarchy on the lattice, it remains however difficult to simulate reliably systems with realistically light pions.

  $\Rightarrow$ more computing power,
  or more clever methods, needed!