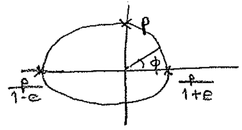
M.L.

Theorie I Klausur 8.4.2009

1

$\alpha > 0, \quad p > 0$ (attraktiv)

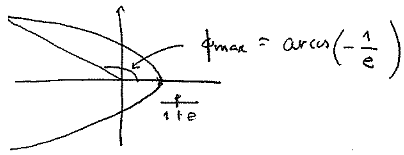
$e < 1$ $\qquad r = \dfrac{p}{1 + e\cos\phi}$



(2pt)

gebundene Bewegung

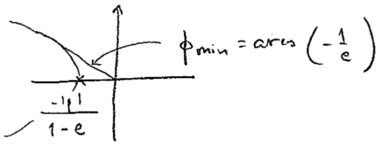
$e > 1$



ungebundene Bewegung (2pt)

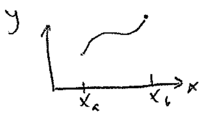
$\alpha < 0, \quad p < 0$ (abstoßend)

$r = \dfrac{-|p|}{1 + e\cos\phi} > 0 \quad \Rightarrow$ nur $e > 1$ möglich



Streuung (2pt)

2



(a) $\quad dl = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1 + [y'(x)]^2}$

$l = \int_{x_a}^{x_b} dx \sqrt{1 + [y']^2}$ (2pt)

(b) $\quad \delta l = \int_{x_a}^{x_b} dx \left\{ f([y'+\delta y']) - f([y']) \right\}; \quad f(y') = \sqrt{1+[y']^2}$

$= \int_{x_a}^{x_b} dx \dfrac{\partial f}{\partial y'} \delta y' + O(\delta y)^2$

$= -\int_{x_a}^{x_b} dx\, \delta y(x) \dfrac{d}{dx}\left(\dfrac{\partial f}{\partial y'}\right) = 0 \quad \Rightarrow \quad \dfrac{d}{dx}\dfrac{\partial f}{\partial y'} = 0 \quad \forall x$

$\Leftrightarrow \quad \dfrac{d}{dx}\left[\dfrac{y'}{\sqrt{1+[y']^2}}\right] = 0$ (4pt)

(c) Aus (b): $\quad y' = $ Konstante

$y' = C$

$\Rightarrow \quad y = Cx + D$ eine gerade Linie $\square$.

(2pt)

(3)

$$\vec{E} = E_0 \vec{e}_2 \left[\Theta(x^1) \cos(kx^1-\omega t) + \Theta(-x^1) \cos(kx^1+\omega t) \right]$$

$$\nabla\times\vec{E} = \vec{e}_3 \partial_1 E^2 = \vec{e}_3 E_0 \left[\delta(x^1) \underbrace{\left(\cos(kx^1-\omega t) - \cos(kx^1+\omega t) \right)}_{=0 \text{ bei } x^1=0} \right]$$

$$- \vec{e}_3 E_0 k \left[\Theta(x^1) \sin(kx^1-\omega t) + \Theta(-x^1) \sin(kx^1+\omega t) \right] \overset{\text{M III}}{=} -\frac{1}{c}\dot{\vec{B}}$$

$$\Rightarrow \quad \vec{B} = \vec{e}_3 E_0 \frac{kc}{\omega} \left[\Theta(x^1) \cos(kx^1-\omega t) - \Theta(-x^1) \cos(kx^1+\omega t) \right] + \text{const.}$$

(2pt)

M II:
$$\vec{J} = \frac{c}{4\pi} \left[\nabla\times\vec{B} - \frac{1}{c}\dot{\vec{E}} \right]$$

$$= \frac{c}{4\pi} \left[-\vec{e}_2 \partial_1 B^3 - \frac{1}{c}\dot{\vec{E}} \right]$$

$$= \frac{c}{4\pi} \Big[-\vec{e}_2 \cdot \frac{E_0 kc}{\omega} \left\{ \delta(x^1) \left(\cos(kx^1-\omega t) + \cos(kx^1+\omega t) \right) \right\}$$
$$+ \vec{e}_2 \frac{E_0 k^2 c}{\omega} \left\{ \Theta(x^1) \sin(kx^1-\omega t) - \Theta(-x^1) \sin(kx^1+\omega t) \right\}$$
$$- \vec{e}_2 \cdot \frac{E_0 \omega}{c} \left\{ \Theta(x^1) \sin(kx^1-\omega t) - \Theta(-x^1) \sin(kx^1+\omega t) \right\} \Big]$$

Benutze: $\omega = kc$!

$$\Rightarrow \quad \vec{J} = -\vec{e}_2 \frac{cE_0}{4\pi} \delta(x^1) \cdot 2 \cdot \cos(\omega t) = -\vec{e}_2 \cdot \frac{cE_0}{2\pi} \delta(x^1) \cos(\omega t)$$

(4pt)

(4)

(a)
$$\frac{dp^i}{d\tau} = \underbrace{\frac{dt}{d\tau}}_{\gamma} \frac{dp^i}{dt} = \frac{q}{c} F^{i\nu} u_\nu = \frac{q}{c} \left(F^{i0} \underbrace{u_0}_{\gamma c} + F^{ij} \underbrace{u_j}_{\gamma v_j} \right)$$

$$\frac{dp^i}{dt} = q \left(\underbrace{F^{i0}}_{E^i} + \frac{1}{c} \underbrace{F^{ij} v_j}_{-\varepsilon^{ijk} B^k v_j = +\varepsilon^{ijk} v^j B^k = (\vec{v}\times\vec{B})^i} \right)$$

(2pt)

(b)
$$\frac{d}{d\tau} p^\mu p_\mu = \frac{d}{d\tau} \eta_{\mu\nu} p^\mu p^\nu = \eta_{\mu\nu} \left(\frac{dp^\mu}{d\tau} p^\nu + p^\mu \frac{dp^\nu}{d\tau} \right)$$

$$= \eta_{\mu\nu} \cdot \frac{q}{c} \cdot \left(F^{\mu\alpha} u_\alpha p^\nu + p^\mu F^{\nu\alpha} u_\alpha \right)$$

$$= \frac{q}{c} \left(F^{\mu\alpha} p_\mu u_\alpha + F^{\mu\alpha} p_\mu u_\alpha \right) = \frac{2q}{c} F^{\mu\alpha} p_\mu u_\alpha \overset{p_\mu = m u_\mu}{=} \frac{2qm}{c} \underbrace{F^{\mu\alpha}}_{\text{antisymmetrisch}} \underbrace{u_\mu u_\alpha}_{\text{Symmetrisch}}$$

$$= 0$$

(2pt)

(c)
$$\frac{d}{d\tau} p^\mu p_\mu = 2 p_\mu \frac{dp^\mu}{d\tau} = -2 p_\mu p^\mu \Gamma = -2\Gamma p^2$$

(1pt)

Aber $p^2 = m^2 u_\mu u^\mu = m^2 c^2 \Rightarrow \frac{dm^2}{d\tau} = -2\Gamma m^2$. (1pt)

Nicht sinnvoll ; Masse bleibt unverändert.