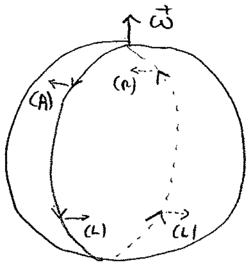
Klausur 1 11.2.2009

1 (a) $m\ddot{x} \neq 0$ auch für $\vec{F} = 0$ ↯ Newton I

(b)



$$\vec{F}_C = 2m\dot{\vec{x}} \times \vec{\omega}$$

$\Rightarrow$ Nordhalbkugel: rechts
Südhalbkugel: Links

2 (a)

$$dl = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1 + [y'(x)]^2}$$

$$V = \int dm\, g\, y = \mu g \int_0^d dx\, \sqrt{1 + [y'(x)]^2}\; y(x)$$

(b)

$$\frac{\partial f}{\partial y} = \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)$$

$$C := y' \frac{\partial f}{\partial y'} - f$$

$$\frac{d}{dx} C = \cancel{y'' \frac{\partial f}{\partial y'}} + y' \underbrace{\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)}_{\frac{\partial f}{\partial y}} - \frac{\partial f}{\partial y} y' - \cancel{\frac{\partial f}{\partial y'} y''} = 0$$

(c)

$$f = \mu g\, y \sqrt{1 + (y')^2}$$

$$\frac{\partial f}{\partial y'} = \mu g\, y \cdot \frac{y'}{\sqrt{1 + (y')^2}}$$

$$y' \frac{\partial f}{\partial y'} - f = \mu g \left\{ \frac{y\,(y')^2}{\sqrt{1+(y')^2}} - \frac{y\,(1 + (y')^2)}{\sqrt{1+(y')^2}} \right\} = \frac{-\mu g\, y}{\sqrt{1+(y')^2}} =: C$$

$$y = a_1 \cosh(a_2 x) \qquad y' = a_1 a_2 \sinh(a_2 x)$$

$$\frac{\mu g\, a_1 \cosh(a_2 x)}{\sqrt{1 + a_1^2 a_2^2 \sinh^2(a_2 x)}} = -C \qquad ; \qquad \cosh^2 - \sinh^2 = 1$$

$$\Rightarrow a_1 a_2 = 1 \qquad \mu g a_1 = -C \qquad a_1 = \frac{-C}{\mu g} \;; \quad a_2 = \frac{-\mu g}{C}$$

$$\Rightarrow \quad y = -\frac{C}{\mu g} \cosh\left(\frac{-\mu g x}{C}\right)$$

(3)

$$\vec{E} = \alpha \vec{e}_1 \cos(\omega t - kz) \qquad \vec{B} = \beta \vec{e}_2 \cos(\omega t - qz)$$

(i) $\nabla \cdot \vec{E} = 0 \qquad \frac{\partial}{\partial x} \cos(\omega t - kz) = 0$ ok.

(ii) $\nabla \cdot \vec{B} = 0 \qquad \frac{\partial}{\partial y} \cos(\omega t - qz) = 0$ ok.

(iii) $\underbrace{\nabla \times \vec{B}} = \underbrace{\frac{1}{c} \dot{\vec{E}}}_{-\frac{\omega}{c} \vec{e}_1 \alpha \sin(\omega t - kz)}$

$$-\vec{e}_1 \frac{\partial}{\partial z} \beta \cos(\omega t - qz) = -\vec{e}_1 \cdot \beta \cdot q \cdot \sin(\omega t - qz) \quad \Rightarrow \quad k = q, \quad \frac{\omega}{c}\alpha = \beta k$$

(iv) $\underbrace{\nabla \times \vec{E}} = \underbrace{-\frac{1}{c} \dot{\vec{B}}}_{\frac{\omega}{c} \vec{e}_2 \beta \sin(\omega t - qz)}$

$$\vec{e}_2 \frac{\partial}{\partial z} \alpha \cos(\omega t - kz) = \vec{e}_2 \alpha k \sin(\omega t - kz) \quad \Rightarrow \quad k = q, \quad \frac{\omega}{c} \beta = \alpha k$$

$$\alpha = \frac{\omega}{c} \frac{\beta}{k}$$

$$\Rightarrow \frac{\omega^2}{c^2} \cdot \frac{1}{k} \cdot \beta = \beta k$$

$$\boxed{\omega = \pm ck} \qquad \boxed{\alpha = \pm \beta} \qquad \boxed{k = q}$$

(4.)

(i) $\quad p_a = \left(\frac{E_a}{c}, \vec{p}_a\right) : \quad p_a^2 = m_a^2 c^2$

$$\Rightarrow m_a^2 = \frac{1}{c^2} p_a^2 = \frac{1}{c^2} (p_b + p_c)^2 = \frac{1}{c^2}\left(p_b^2 + p_c^2 + 2 p_b \cdot p_c\right)$$

$$= m_b^2 + m_c^2 + \frac{2}{c^2}\left(\frac{E_b E_c}{c^2} - \vec{p}_b \cdot \vec{p}_c\right)$$

(ii) $\quad \vec{p}_a = \vec{p}_b + \vec{p}_c \quad ; \quad E_a = E_b + E_c = c\sqrt{m_b^2 c^2 + \vec{p}_b^{\,2}} + c\sqrt{m_c^2 c^2 + \vec{p}_c^{\,2}}$

$$\Rightarrow m_a^2 c^2 = \frac{E_a^2}{c^2} - \vec{p}_a^{\,2} = \frac{(E_b + E_c)^2}{c^2} - (\vec{p}_b + \vec{p}_c)^2$$

$$= \underbrace{\frac{E_b^2}{c^2} + \frac{E_c^2}{c^2} - \vec{p}_b^{\,2} - \vec{p}_c^{\,2}}_{m_b^2 c^2 + m_c^2 c^2} + 2\left(\frac{E_b E_c}{c^2} - \vec{p}_b \cdot \vec{p}_c\right)$$

(i) oder (ii):

$$m_a^2 = m_b^2 + m_c^2 + \frac{2}{c^2}\left[\sqrt{m_b^2 c^2 + \vec{p}_b^{\,2}}\sqrt{m_c^2 c^2 + \vec{p}_c^{\,2}} - \vec{p}_b \cdot \vec{p}_c\right]$$