Probeklausur 20.07.2010 EMTP II M.L.

1 (a) $\vec r = (r\sin\theta\cos\varphi,\ r\sin\theta\sin\varphi,\ r\cos\theta)$; $h_r = |\partial_r \vec r| = 1$, $h_\theta = |\partial_\theta \vec r| = r$, $h_\varphi = |\partial_\varphi \vec r| = r\sin\theta$

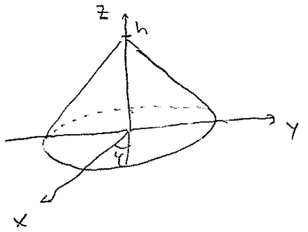
$$\nabla = \vec e_r \partial_r + \frac{\vec e_\theta}{r}\partial_\theta + \frac{\vec e_\varphi}{r\sin\theta}\partial_\varphi$$

$$\Rightarrow \nabla\Phi = -\frac{2\cos\theta\,\vec e_r}{ar^3} - \frac{\sin\theta\,\vec e_\theta}{ar^3}$$

(b) $\vec r = (\rho\cos\varphi,\ \rho\sin\varphi,\ z)$; $h_\rho = |\partial_\rho \vec r| = 1$, $h_\varphi = |\partial_\varphi \vec r| = \rho$, $h_z = |\partial_z \vec r| = 1$

$$\Rightarrow \nabla\times\vec v = \frac{1}{\rho}\begin{vmatrix} \vec e_\rho & \rho\vec e_\varphi & \vec e_z \\ \partial_\rho & \partial_\varphi & \partial_z \\ 0 & \rho^2+k & 0 \end{vmatrix} = \frac{1}{\rho}\cdot\vec e_z\,\partial_\rho(\rho^2+k) = 2\vec e_z$$

2



Benutze Zylinderkoordinaten; $0\le\rho\le h$; $z = h-\rho$

$$\vec r = (\rho\cos\varphi,\ \rho\sin\varphi,\ h-\rho)$$

$$\partial_\rho \vec r = (\cos\varphi, \sin\varphi, -1)$$

$$\partial_\varphi \vec r = (-\rho\sin\varphi, \rho\cos\varphi, 0)$$

$$\partial_\rho \vec r\times\partial_\varphi\vec r = \begin{vmatrix} \vec e_\rho & \vec e_\varphi & \vec e_z \\ \cos\varphi & \sin\varphi & -1 \\ -\rho\sin\varphi & \rho\cos\varphi & 0 \end{vmatrix} = \dots + \vec e_z(\rho\cos^2\varphi + \rho\sin^2\varphi) = \rho\vec e_z$$

$$\text{Fluß} = \int_0^{2\pi} d\varphi \int_0^h d\rho\ \partial_\rho\vec r\times\partial_\varphi\vec r\cdot\vec B = 2\pi\cdot B\cdot\int_0^h d\rho\,\rho = \pi B\cdot h^2$$

! ⇒ Aber viel einfacher: $\nabla\cdot\vec B = 0$ ⇒ kann auch die untere Decke benutzen ⇒ Fluß $= B\cdot\underbrace{\pi h^2}_{\text{Fläche}}$

3 $\vec E = q\frac{\vec r}{r^3} = \frac{q}{r^2}\vec e_r$

(a) $$\nabla\times\vec E = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec e_r & r\vec e_\theta & r\sin\theta\,\vec e_\varphi \\ \partial_r & \partial_\theta & \partial_\varphi \\ \frac{q}{r^2} & 0 & 0 \end{vmatrix} = 0$$

(b) $$-\nabla\Phi = -\left\{\vec e_r\partial_r\Phi + \frac{\vec e_\theta}{r}\partial_\theta\Phi + \frac{\vec e_\varphi}{r\sin\theta}\partial_\varphi\Phi\right\} \stackrel{!}{=} \frac{q}{r^2}\vec e_r \Rightarrow \partial_r\Phi = -\frac{q}{r^2} \Rightarrow \Phi = \frac{q}{r}$$

(c) $$\oint_S d\vec A\cdot\vec E = \int_V dV\,\nabla\cdot\vec E = \int_V dV\left(-\nabla\cdot\nabla\frac{q}{r}\right) = -q\int_V dV\,\nabla^2\frac{1}{r} = -q\int_V dV\left(-4\pi\delta^{(3)}(\vec r)\right) = 4\pi q$$

4 (a) $$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i\frac{2\pi n x}{L}} \qquad f^2(x) = \sum_{n,m} c_n c_m e^{i\frac{2\pi(n+m)x}{L}}$$

$$\frac{1}{L}\int_{-L/2}^{L/2} dx\,[f(x)]^2 = \sum_{n,m} c_n c_m \underbrace{\frac{1}{L}\int_{-L/2}^{L/2} dx\, e^{i\frac{2\pi(m+n)x}{L}}}_{\substack{1,\ m=-n \\ 0,\ m\neq -n}} = \sum_{n=-\infty}^{\infty} c_n c_{-n} = c_0^2 + 2\sum_{n=1}^{\infty} c_n c_{-n}$$

$f(x)$ reell ⇒ $c_{-n} = c_n^*$ ⇒ □.

(b) $$c_n^* = \frac{1}{L}\int_{-L/2}^{L/2} dx\, f(x)\, e^{i\frac{2\pi n x}{L}} \ ; \quad c_n = \underbrace{\frac{1}{L}\int_{-L/2}^{L/2} dx\, f(x)\, e^{-i\frac{2\pi n x}{L}}}_{x\to -x} = \frac{1}{L}\int_{-L/2}^{L/2} dx\, f(-x)\, e^{i\frac{2\pi n x}{L}}$$

⇒ $f(x) = f(-x)$ (symmetrisch)

eigentlich „hinreichend"